Synchronization Phenomena & Chimera States in Coupled Oscillator Networks

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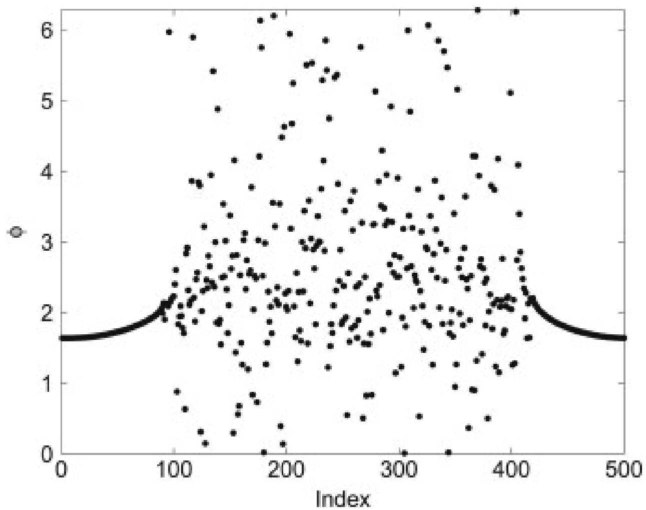


26th Summer School-Conference on
“Dynamical Systems and Complexity”
NTUA, Athens 2019

Overview:

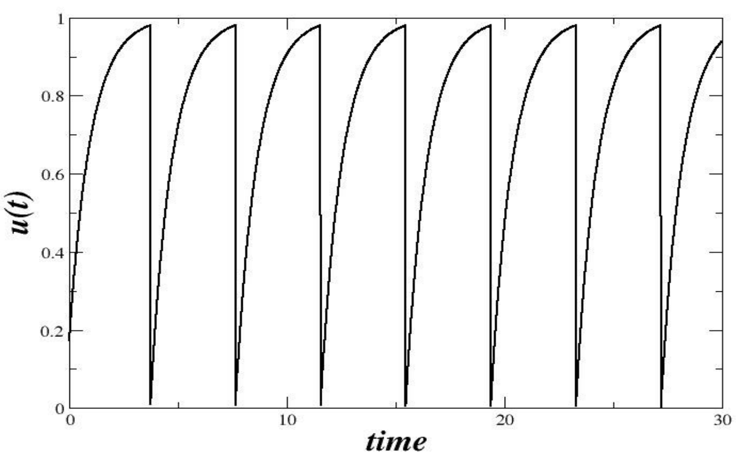
1. Introduction & Motivation
   - The System: Network and Dynamics
   - Dynamics and Synchronization phenomena:
   - What is a chimera state?
   - Applications in Brain Science et al.
2. The Leaky Integrate-and-Fire (LIF) Model
   - Nonlocal Connectivity
   - Other connectivities (Reflecting, Diagonal)
   - Hierarchical Connectivity
   - Non-local connectivity 2D & 3D
3. The FitzHugh Nagumo (FHN) Model
   - Non-local Connectivity 1D
   - Hierarchical Connectivity
4. Conclusions & Open Problems

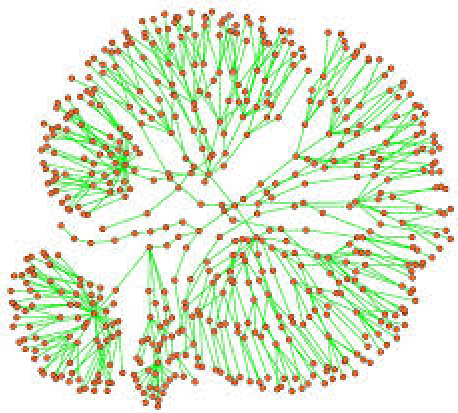
1.1 Coupled Networks and Dynamics

Ingredients

Non-linear Oscillators

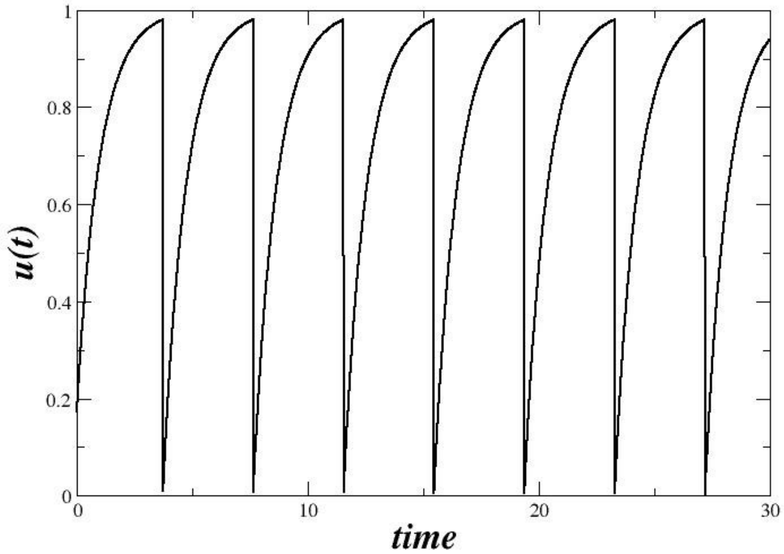
Complex Network Connectivity





$$\frac{du_i(t)}{dt} = f[u_i(t)] + \sum_{j=1}^{N} \sigma_{ij}[u_j(t) - u_i(t)]$$

1.4 Dynamics of Single Oscillators & Synchronization phenomena



Single Element

!!!Spiking!!!

Coupled system

- **Single frequency!!! or**
- Distribution of frequencies and/or
- Distribution of parameters and/or
- Distribution of coupling constants

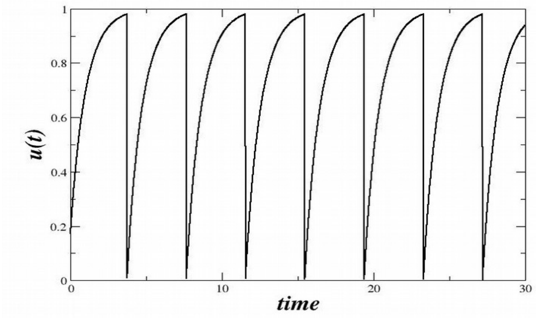
Typical models of nonlinear !**neuron**! oscillators

Leaky Integrate-and-Fire Model
(Louis Lapicque, 1907)

$$\frac{du(t)}{dt} = \mu - u(t)$$

$$u(t) \rightarrow 0, \quad when \quad u(t) > u_{th}$$

2 parameters

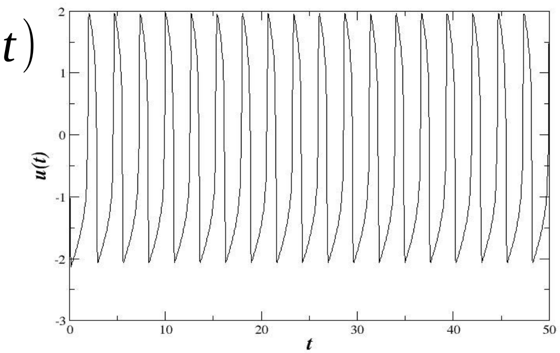


FitzHugh-Nagumo Model
(1961-1962)

$$\epsilon \frac{du(t)}{dt} = u(t) - \frac{u^3(t)}{3} - v(t) + I(t)$$

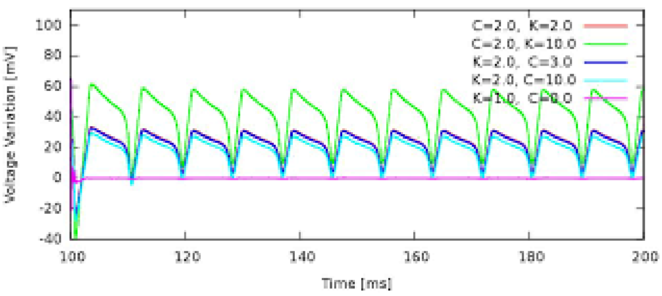
$$\frac{dv(t)}{dt} = u(t) + \alpha$$

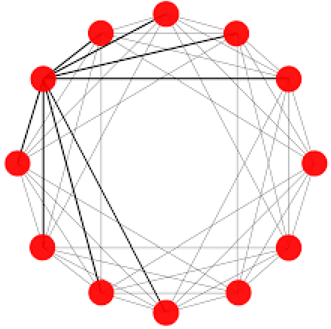
2 parameters



Hodgkin-Huxley Model
(1952)

5 equations
16 parameters



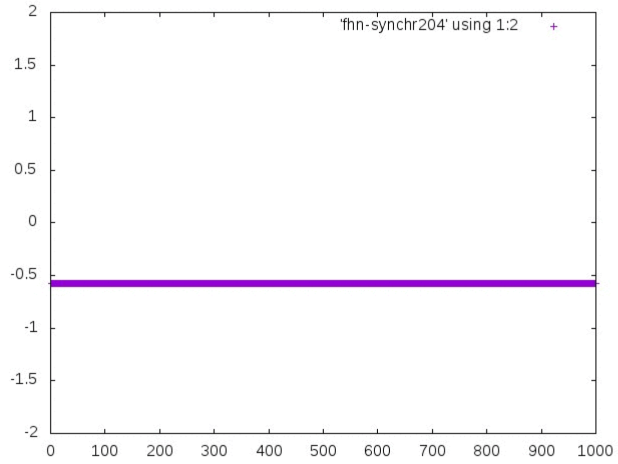


1.5 Synchronization phenomena

1. Full synchronization:
   Starting from random initial states

   $u_i(t=0) \neq u_j(t=0)$, $i,j=1,2,...N$,

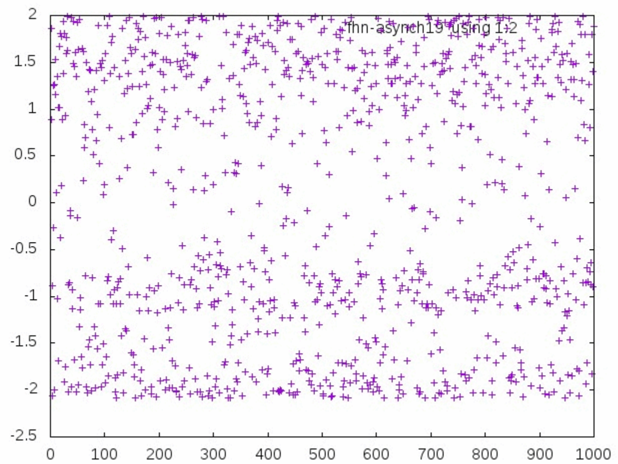
   $\exists\, t_0 : u_i(t) = u_j(t) \;\; \forall t \;\; \& \;\; \forall (i,j)$, for $t>t_0$



2. No-synchronization:
   Starting from random initial states

   $u_i(t=0) \neq u_j(t=0)$, $i,j =1,2 \ldots N$,

   $\Rightarrow u_i(t) \neq u_j(t) \;\; \forall t$ and $\forall (i,j)$



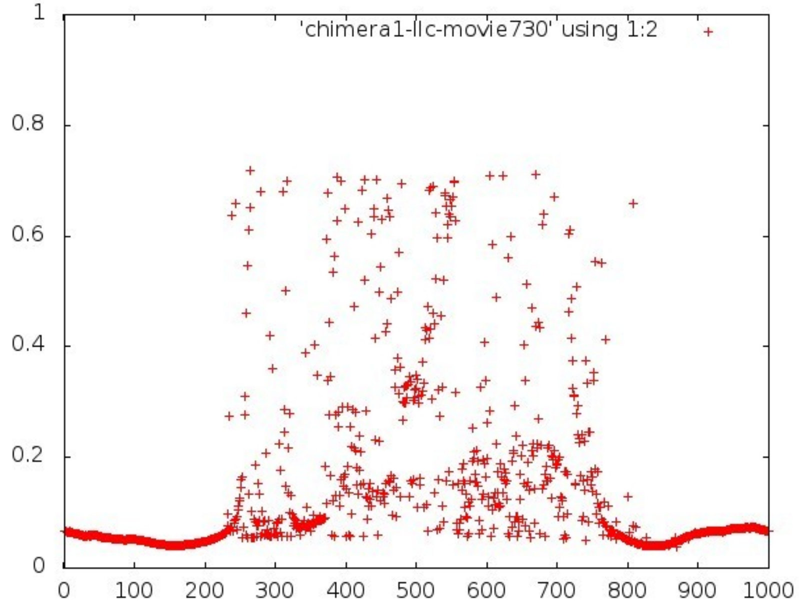
3. Partial synchronization:
Starting from random initial states
and identical oscillators & $\sigma_{ij} = \boldsymbol{\sigma}$

$$u_i(t=0) \neq u_j(t=0),\ i, j = 1, 2 \ldots N$$

$$\forall\ t_0\ \&\ \{K\}=[K, K+1, K+2, \ldots, K+K']$$
$$: u_l(t) = u_m(t) \quad \forall\ (l,m) \in \{K\},\ for\ t>t_0$$

while

$$u_i(t) \neq u_j(t) \quad \forall\ (i,j) \notin \{K\},\ for\ t>t_0$$



??? Partial Brain Activity ???
???Fundamental Understanding of Chimeras???

1. 6 Elements of Chimera States

(Abrams and Strogatz in 2004)

Elements:
- identical oscillators
- identically linked in networks
- random initial conditions

Outcomes:

*Complete Synchronization

++ Partial synchronization
(or partial disorder...)
"Chimera State"

*Complete disorder

Chimera monster: with head of a lion, body of a goat, and tail of a snake.

Red-figure Apulian plate, c. 350–340 BC

-2002: Kuramoto and Battogtokh, Nonlin. Phen. in Complex Sys., 5:380.

-2004: Abrams and Strogatz, Phys. Rev. Lett., 93:174102.

-2015: Panaggio and Abrams, Nonlinearity, 28:R67 (review).

-2016: Schöll, EPJ-ST, 225:891 (review).

-2018: Omel'chenko, Nonlinearity, 31 (5), R121 (review).

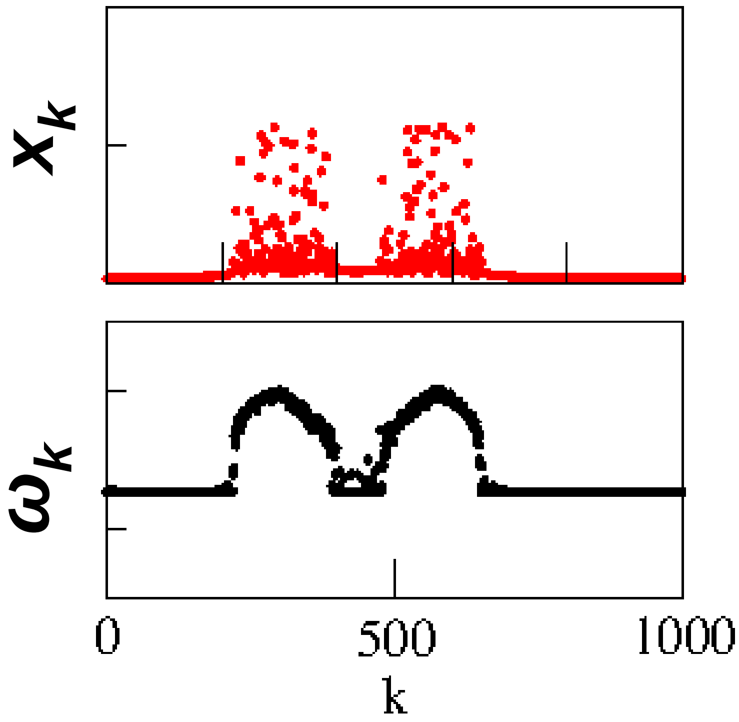
Quantitative Description

$$\omega_i = \frac{Number\ of\ cycles\ of\ element\ i\ \mathrm{in}\ time\ \Delta T}{2\pi \quad \Delta T}$$

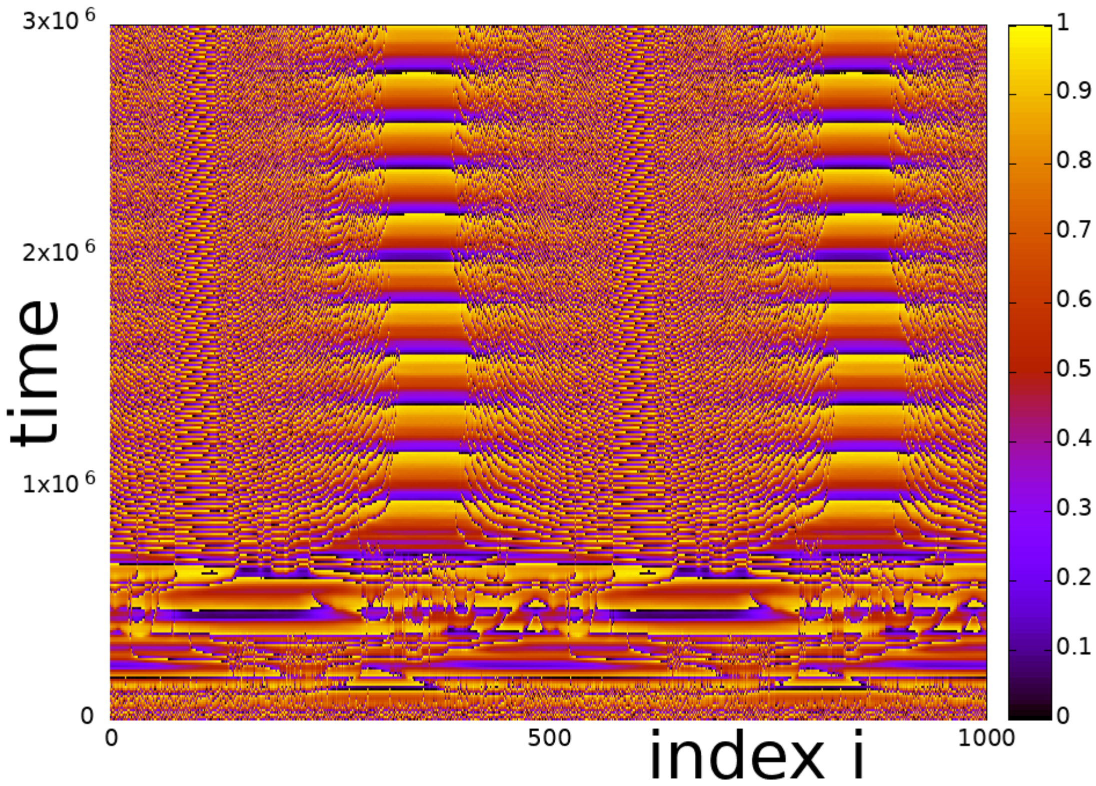
$$\Delta\omega = \omega_{incoh} - \omega_{coh}$$

$$N_{incoh} = \frac{1}{N}\sum_{i=1}^{N} \Theta\left(\omega_i - \omega_{coh} - c\right)$$

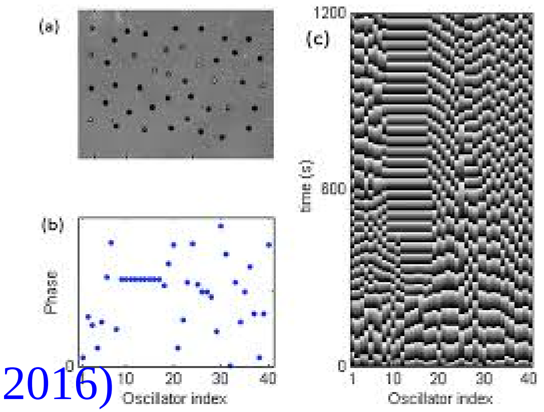
$$M_{incoh} = \sum_{i=1}^{N} \left(\omega_i - \omega_{coh}\right)$$



Visual representation via the space-time plot



1.7 Experiments



Now it has experimental verifications in the domains:
*Mechanics: Coupled metronomes
(*Martens et al, Proc. Nat. Acad. Sciences, 2013*)
(Blaha, Burrus,… Sorrentino, Chaos, 2017)
*Electronics: Equivalent circuits
(Meena et al., Int. Jour. Bifurcations and Chaos, 2016)
(Klinshov … Nekorkin, Phys. Rev. E, 2016)
*Chemical Dynamics: BZ experiments
(*Tinsley …. Showalter, Nature Physics, 2012*),
(Taylor … Showalter, Phys.Chem.ChemPhys. 2016).

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*Lasers: Optical coupled-map lattices via liquid-crystal spatial light modulators
(*Hagerstrom et al., Nature Physics, 2012*)
(Viktorov, Habruseva, ...Kelleher, CLEO-IQEC-2013).

*Uni-hemispheric sleep in birds and dolphins (*Panaggio and Abrams, 2015*)
* Partial & mal-functionality of the brain-Epilepsy (Mormann et al, *2012, Anderjack et al., 2016*)
* Synchronization phenomena in the firing of fireflies etc *(Ott, Antonsen, Chaos 2017)*

Videos:
https://www.quantamagazine.org/physicists-discover-exotic-patterns-of-synchronization-20190404/
https://www.youtube.com/watch?v=_3q6ni6C0Z4

1.8 Applications in Neuron Dynamics

Partial Synchronisation in the form of Chimera States is first numerically observed in the domain of neuron dynamics:

* Phase Oscillator (*Kuramoto et al. 2002, Abrams et al. 2004*)
* FitzHugh Nagumo Oscillator (*Omelchenko et al, 2013, 2014, 2015*)
* Leaky Integrate-and-Fire (*Olmi et al., 2010, Luccioli et al. 2010, Tsigkri et al. 2015*)
* van der Pol oscillators (*Ulonska et al., 2016*)
* Hindmarsh-Rose Oscillator (*Hizanidis et al., 2014, 2016*) ……..

Population Dynamics & Reaction Diffusion:
* BZ Reaction: (T*insley …. Showalter, Nature Physics, 2012)*
* Population Dynamics *(Hizanidis …, PRE 2015)*

Materials:
* Metamaterials: *(Lazarides et al., 2015; Hizanidis et al. 2016; Shena et al. 2017)*

Importance & Influence of :

a) Dynamics	b) Network Topology
Spiking	Nonlocal Connectivity
Cut-offs	Topology of connections
System parameters	Coupling strength

2.1 The Leaky Integrate-and-Fire Model (Louis Lapicque, 1907)

[propagation of electrical signals in neurons, simple, popular in computational neuroscience]

$$\frac{du(t)}{dt} = \mu - u(t)$$

$$u(t) \rightarrow u_{rest}, \quad when \quad u(t) > u_{th}$$

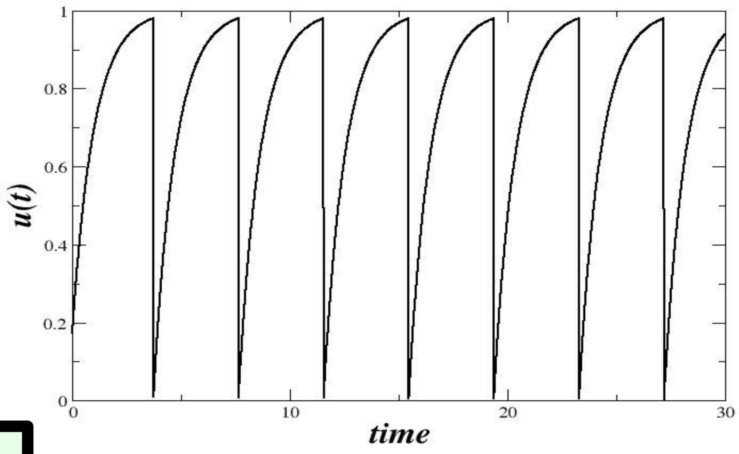
$u(t)$=membrane potential

p_r=refractory period

μ= leaky integrator constant

$$u(t) = \mu - (\mu - u_{rest})e^{-t}$$

$$for \quad u_{rest} < u(t) < u_{th}$$

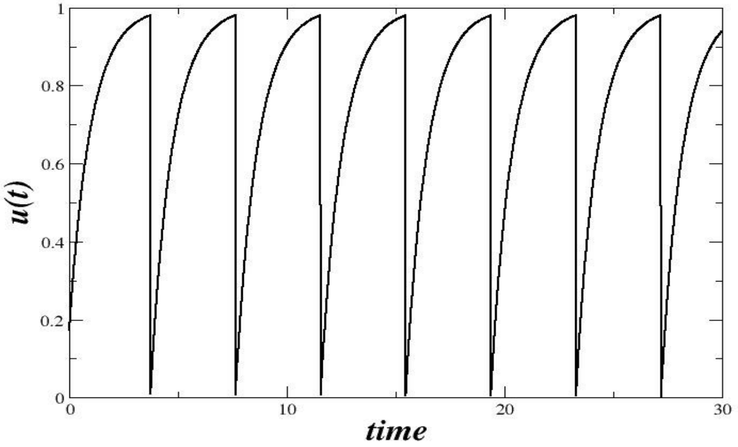


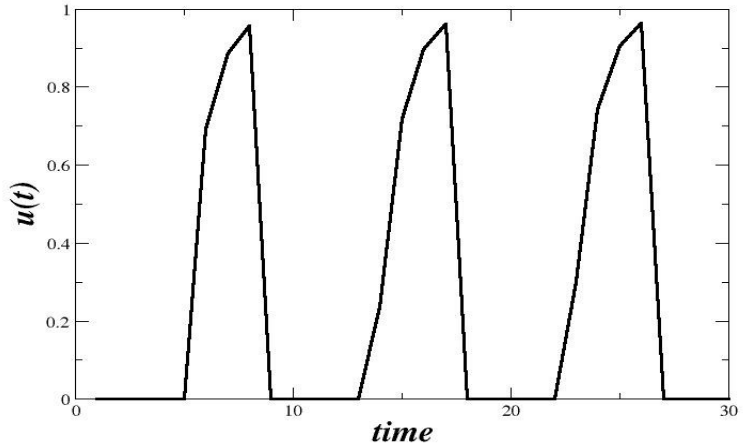
$p_r = 0$

LIF Model with refractory period

$$T = \ln \frac{\mu - u_{rest}}{\mu - u_{th}}$$

$$T = \ln \frac{\mu - u_{rest}}{\mu - u_{th}} + p_r$$





$p_r = 0$

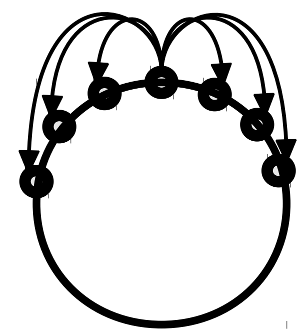
$p_r \neq 0$

2.2 Coupled LIF oscillators in various connectivity schemes

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{1}{R} \sum_{j=connect.} \sigma_{ij} [u_i(t) - u_j(t)]$$

$$u_i(t) \rightarrow 0, \quad when \quad u_i(t) > u_{th}$$

non-local



σ_{ij} = coupling strength, [σ , 0]

μ =1

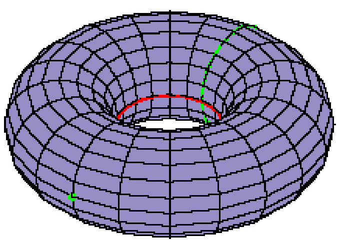
u_{th}=0.98

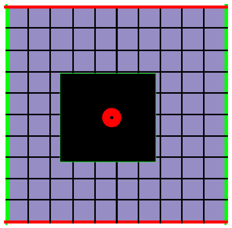
*Periodic boundary conditions:

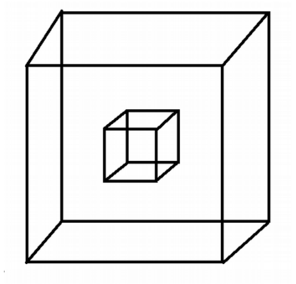
1D-> ring

2D-> torus

3D-> hypertorus





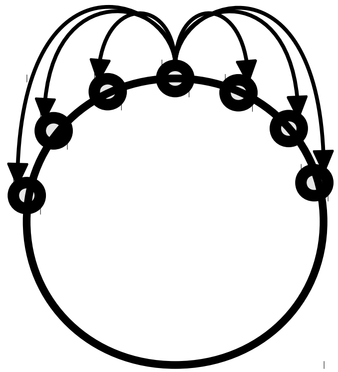


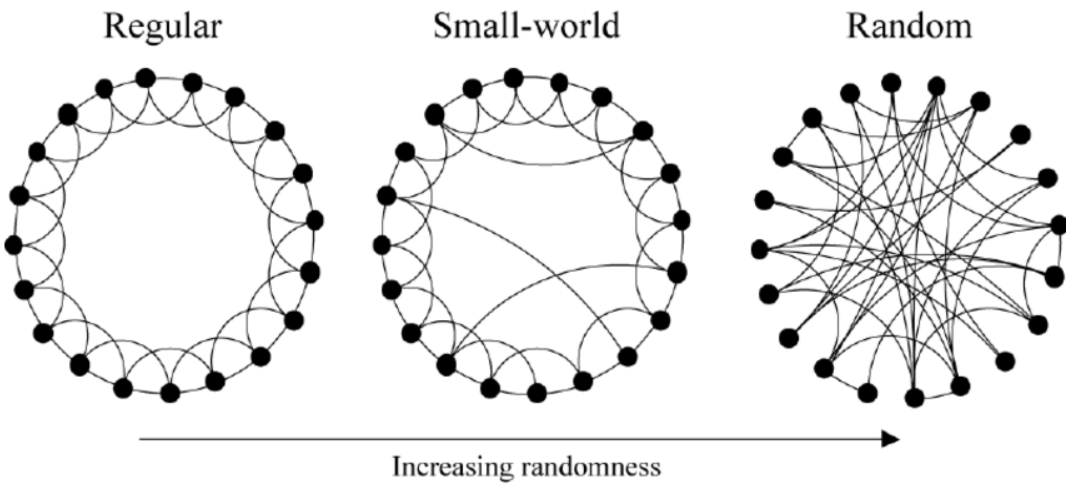
*Variables: σ, p_r, geometry

2.2 Coupled LIF oscillators…. (continued)

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{1}{R} \sum_{j=connect.} \sigma_{ij} [u_j(t) - u_i(t)]$$

$$u_i(t) \to 0, \quad when \quad u_i(t) > u_{th}$$





σ_{ij} = coupling strength, μ =1, u_{th}=0.98, N=1000
*Periodic boundary conditions on a ring
*Variables: σ, p_r, geometry

Olmi, Politi & Torcini, EPL, vol. 92, 60007 (2010)
Luccioli & Politi, PRL, vol. 105, 158104 (2010)

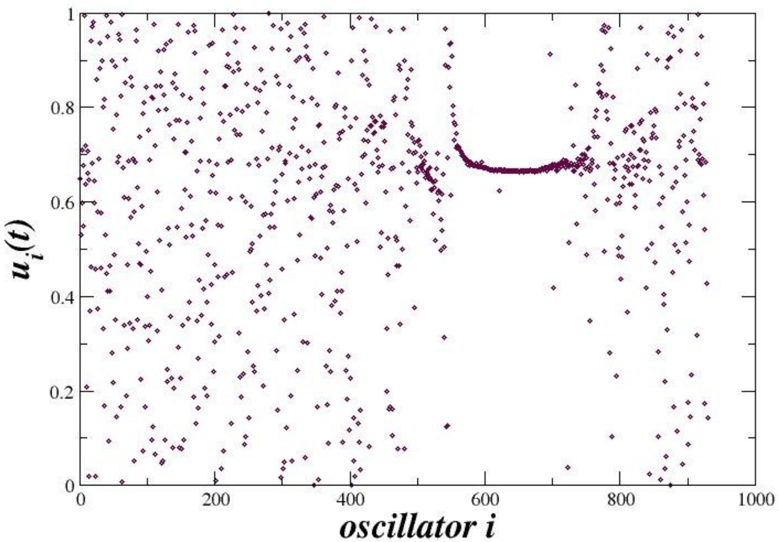
2.3 Coupled LIF Oscillators in 1D (ring)

$$\frac{du_i(t)}{dt} = \mu - u_i(t) + \frac{\sigma}{2R} \sum_{j=i-R}^{i+R} [u_i(t) - u_j(t)]$$

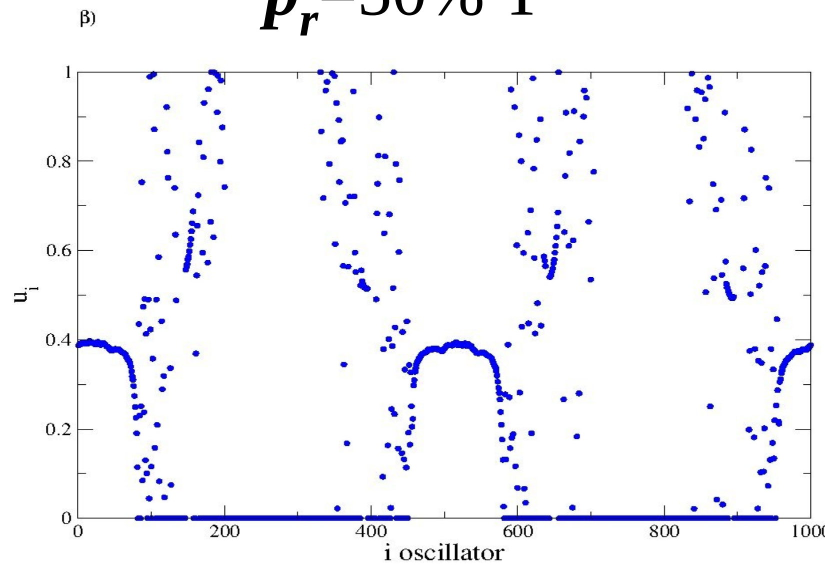
$$u_i(t) \rightarrow 0, \quad when \quad u_i(t) > u_{th}$$

σ=0.656
R=350

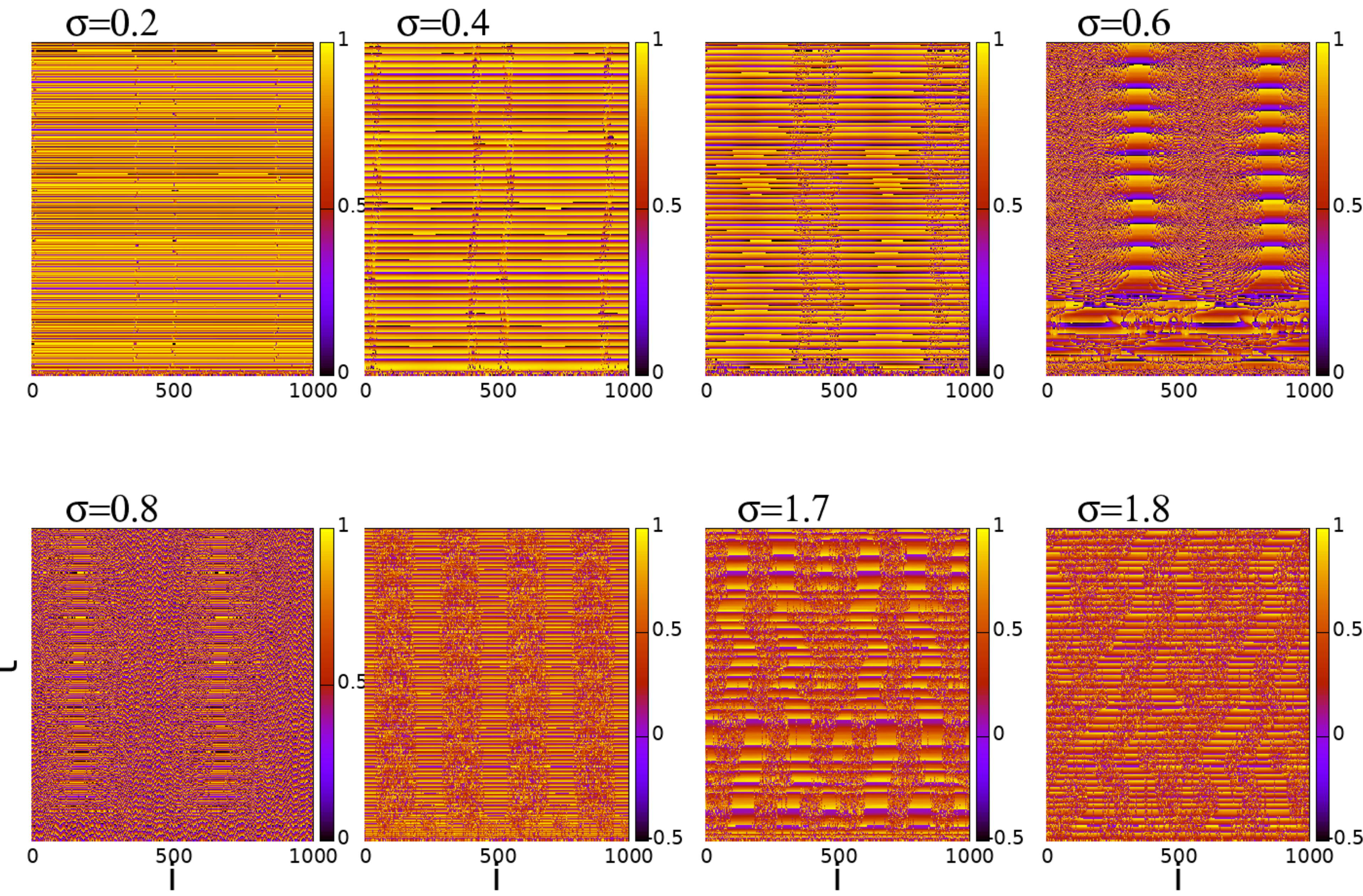
a) Without refractory period
=> single chimera
p_r=0



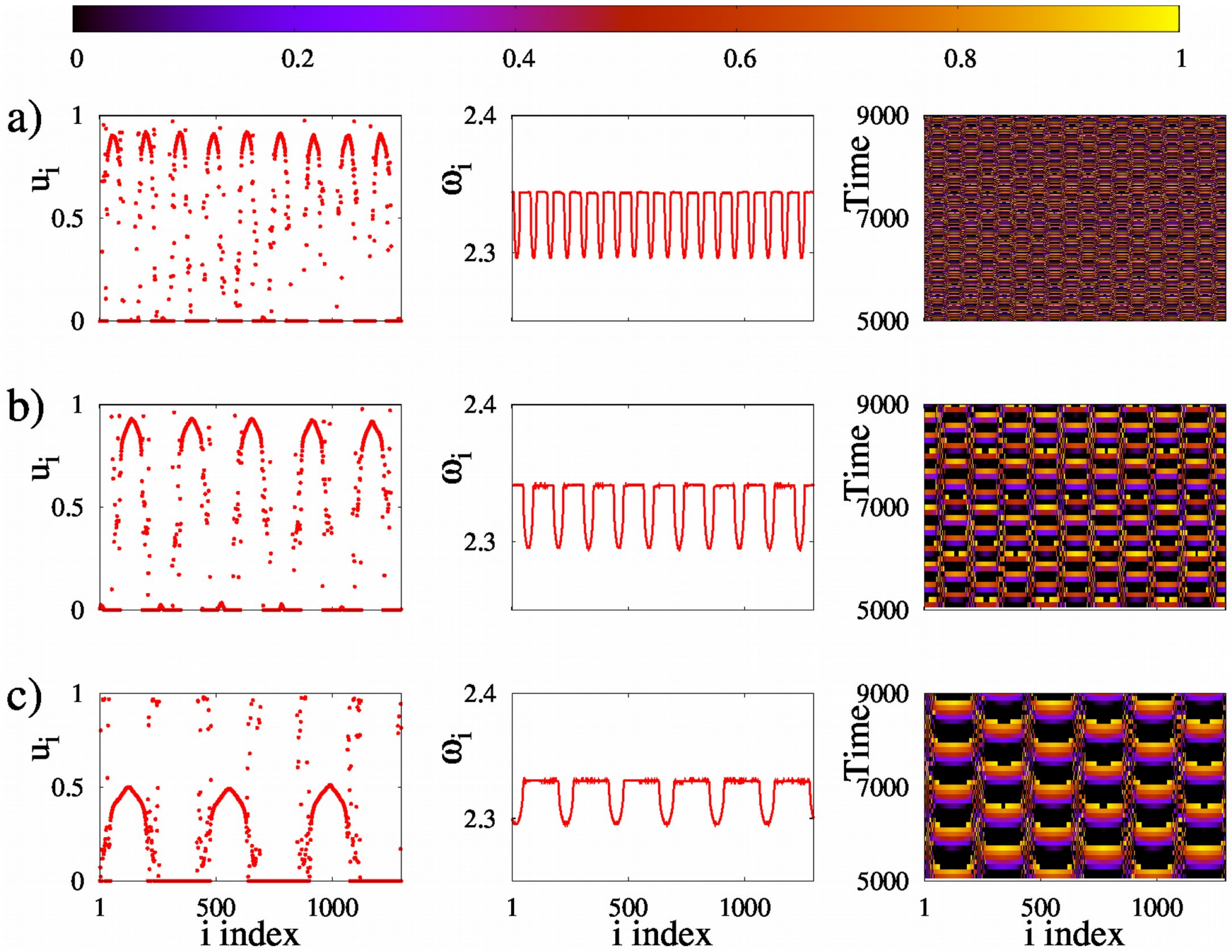
b) With refractory period
=> multi-chimera
p_r=50% T



Coupled LIF Oscillators: Influence of coupling strength



σ=0.7, p_r=30%



R=100

R=175

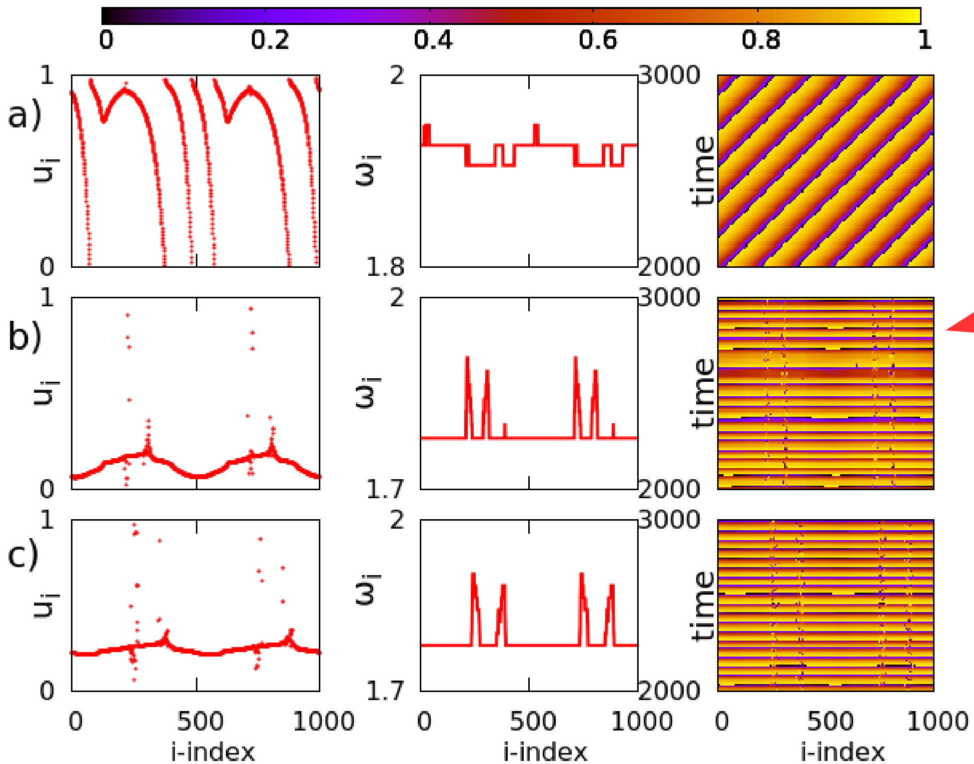
R=350

As $R\uparrow$ the number of (in)coherent parts decreases: Expected...

Parameter range for chimeras : $\sigma \in (0.5\ , 0.8)$, $p_r \in (0T_s\ , 1.0T_s)$

2.4 Reversion of coherence

2.4.1 Solitaries/Chimeras for small σ < 1.0



a) R = 10 (d=0.041),

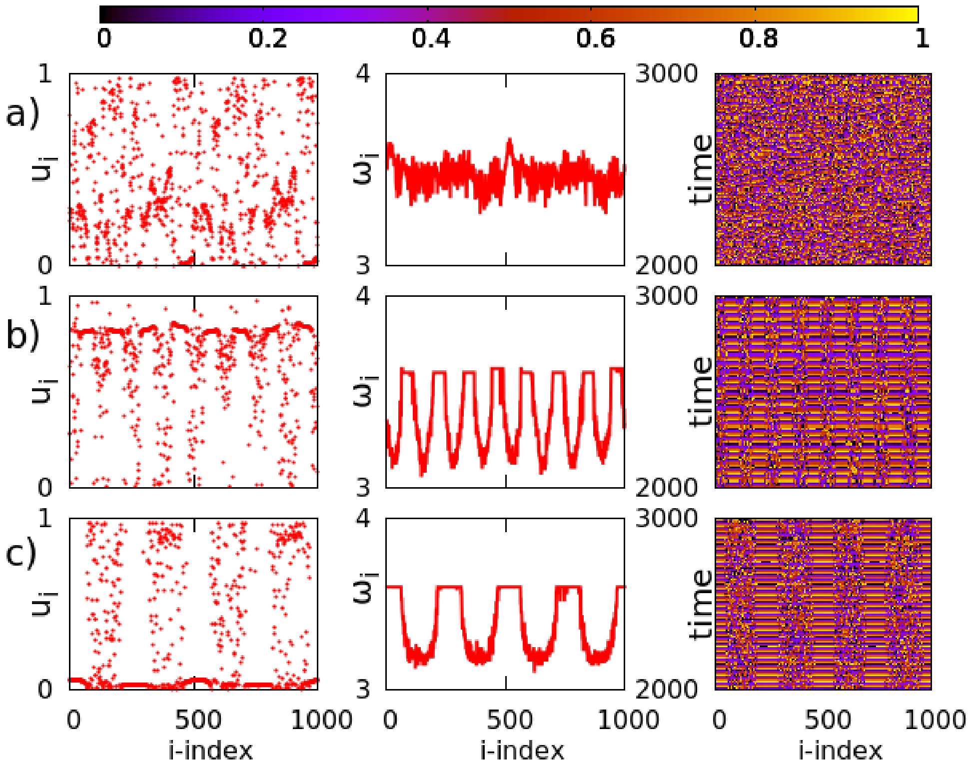
instability

b) R = 100 (d=0.401)

c) R = 150 (d=0.601).

Other parameters are: σ = 0.4, N = 1000, μ = 1. and u_{th} = 0.98

2.4.2 Typical Chimeras σ>1.5



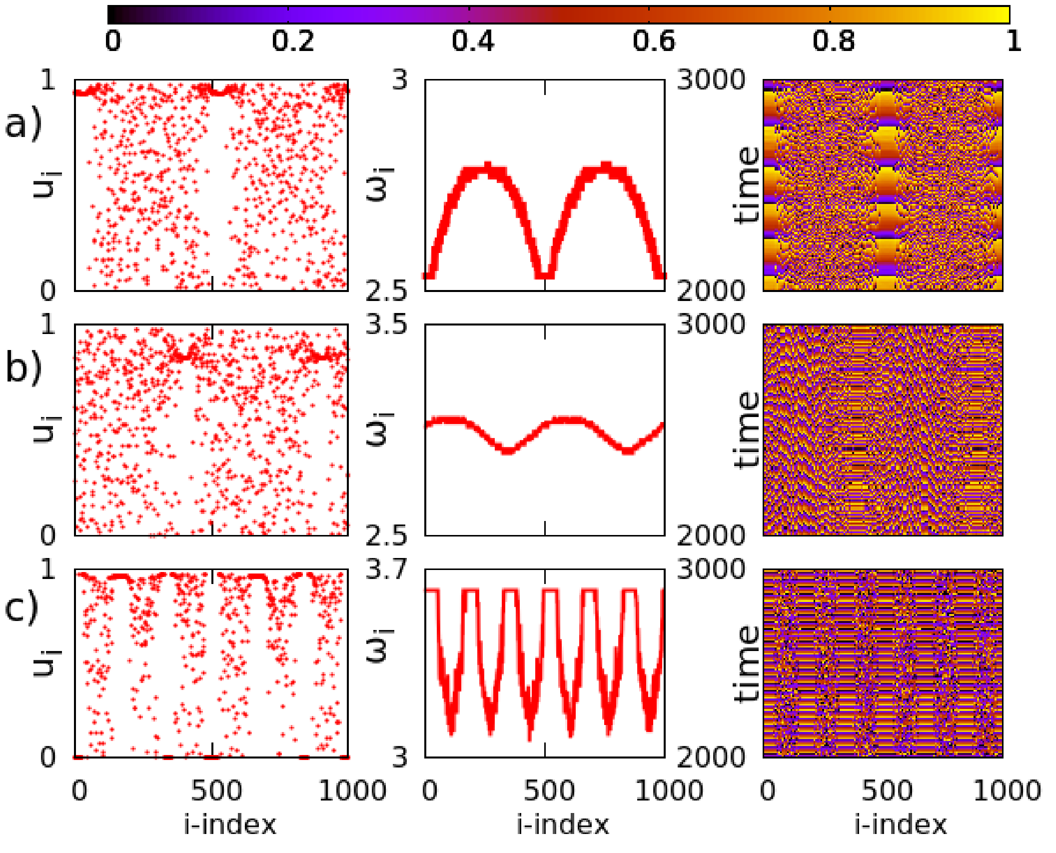
a) R = 10 (d=0.041),

b) R = 100 (d=0.401)

c) R = 150 (d=0.601).

Other parameters are: σ = 1.6, N = 1000, μ = 1. and u_{th} = 0.98

2.4.3 Coherence reversion with σ



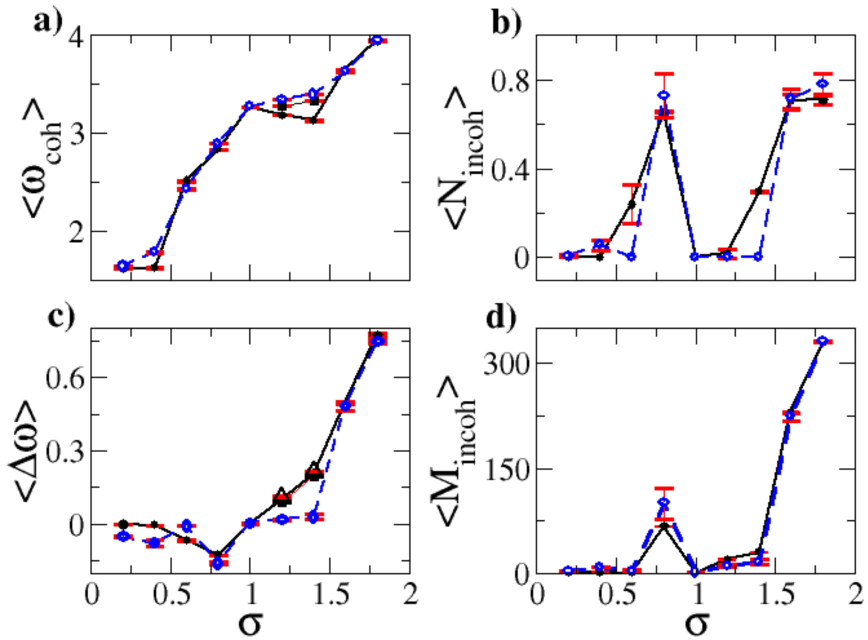
σ = 0.6

σ = 0.8

σ = 1.6

Other parameters are: R= 120 (d=0.481), N = 1000, μ = 1. and u_{th} = 0.98

2.4.4 Dependence on σ (continued)

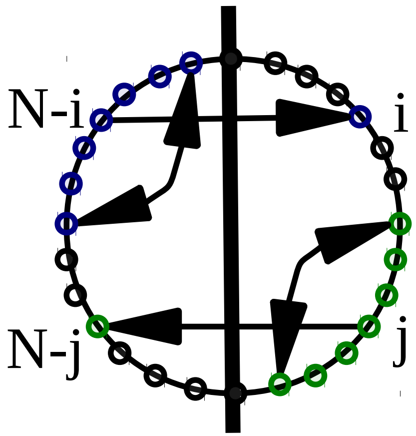


Transition is shown between

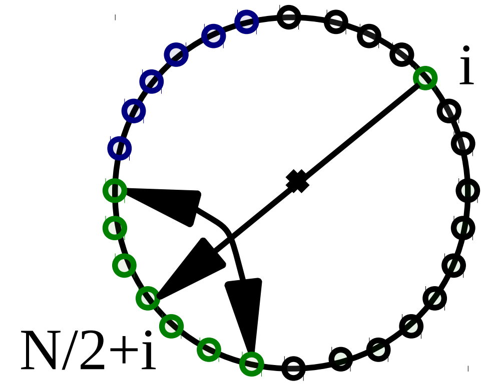
1.0<σ<1.5

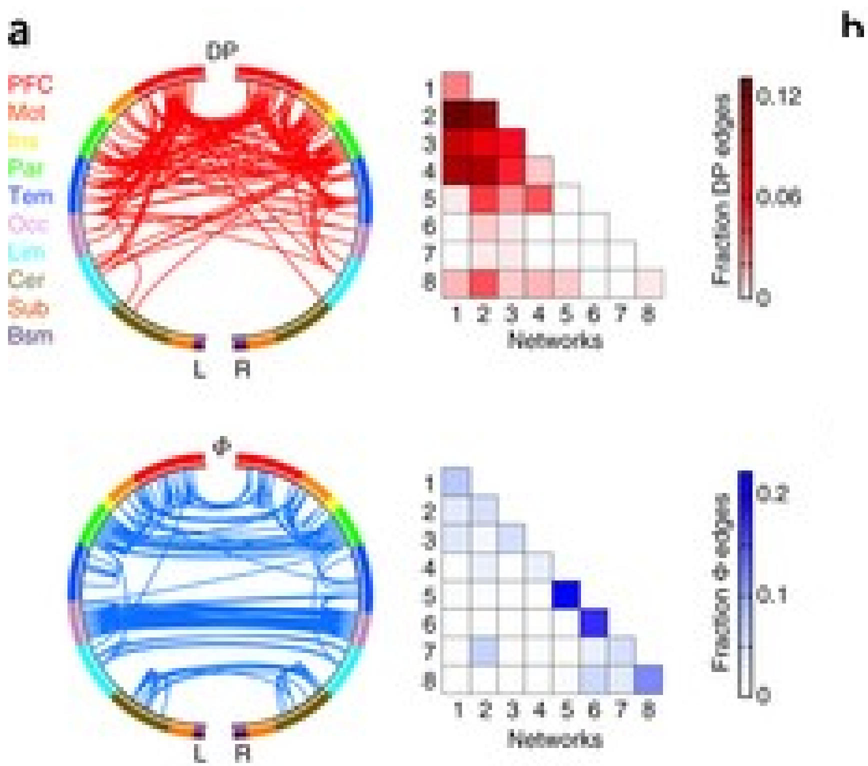
Parameters are: R= 120 (d=0.481)-(blue-dashed lines)
R= 200 (d=0.801)-(black-solid lines),
N = 1000, μ = 1. and u_{th} = 0.98

2.4 Coupled LIF oscillators in various connectivity schemes



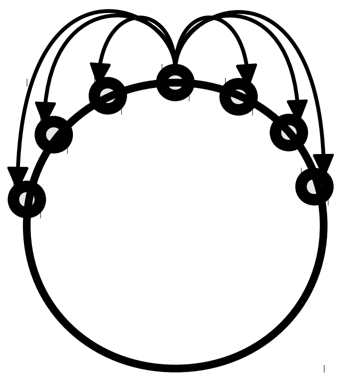
1D
Reflecting
Coupling
and
Diagonal
Coupling





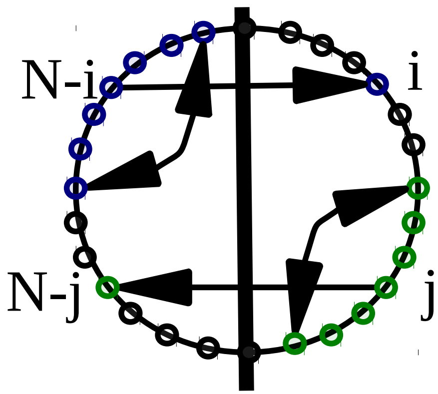
Finn et al.,
Nature Neuroscience,
Vol. 18, p. 1664 (2015)

2.4 Coupled LIF ... connectivity schemes (continued)



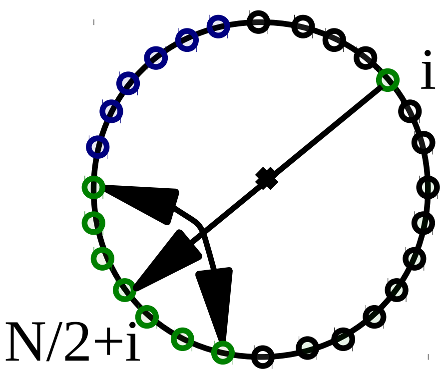
Non-local connectivity

$$\sigma_{ij} = \begin{cases} \sigma & \text{if} \quad N-i-R \leq j \leq N-i-R \\ 0 & \text{otherwise} \end{cases}$$



Reflecting connectivity

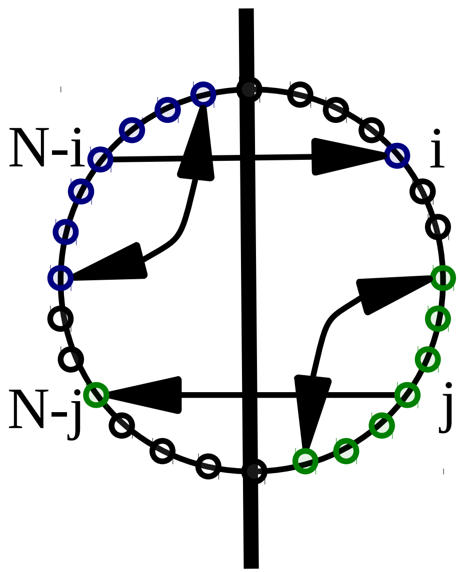
$$\sigma_{ij} = \begin{cases} \sigma & \text{if} \quad N-i-R \leq j \leq N-i-R \\ 0 & \text{otherwise} \end{cases}$$

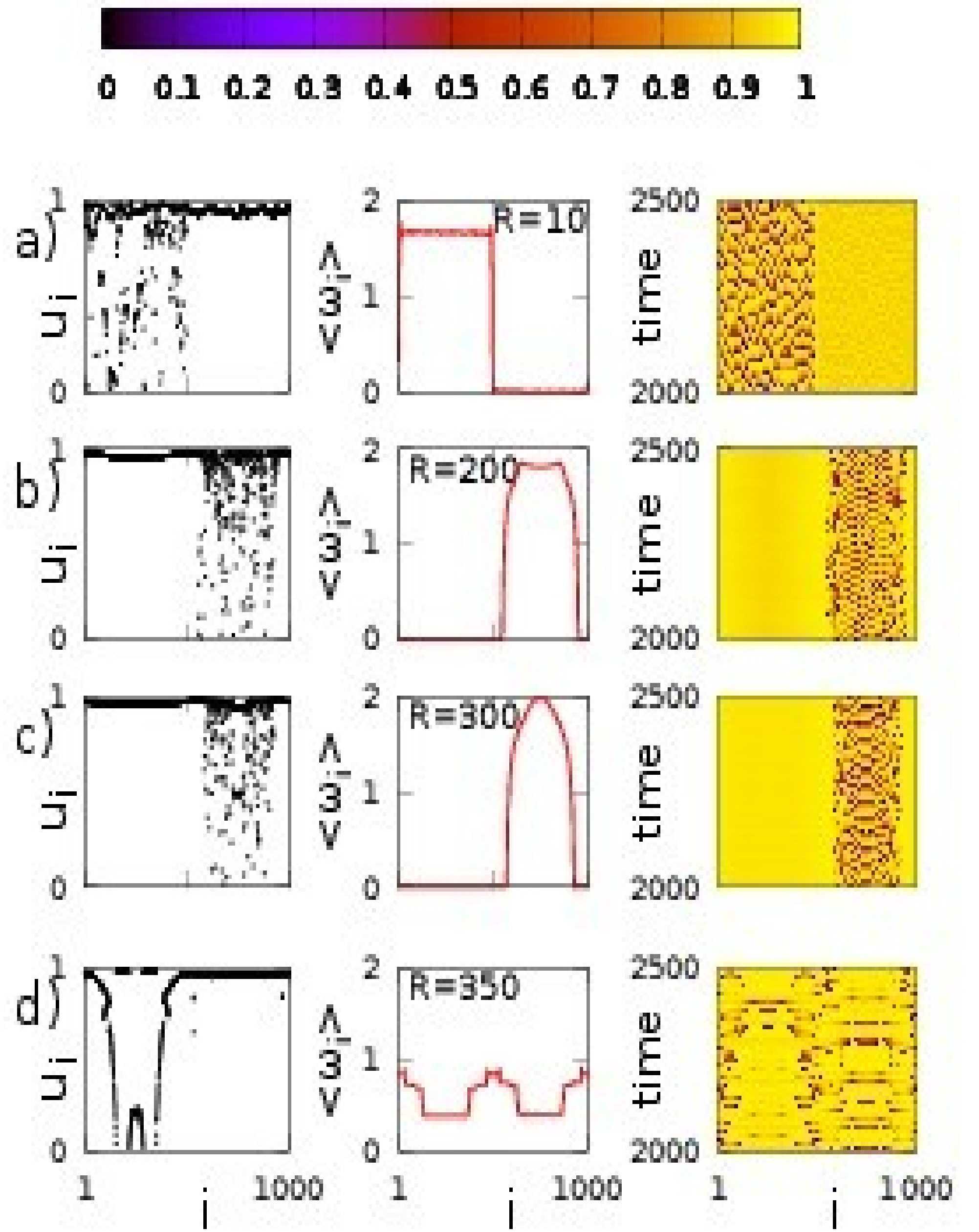


Diagonal connectivity

$$\sigma_{ij} = \begin{cases} \sigma & \text{if} \quad \frac{N}{2}+i-R \leq j \leq \frac{N}{2}+i-R \\ 0 & \text{otherwise} \end{cases}$$

2.5 Reflecting Connectivity

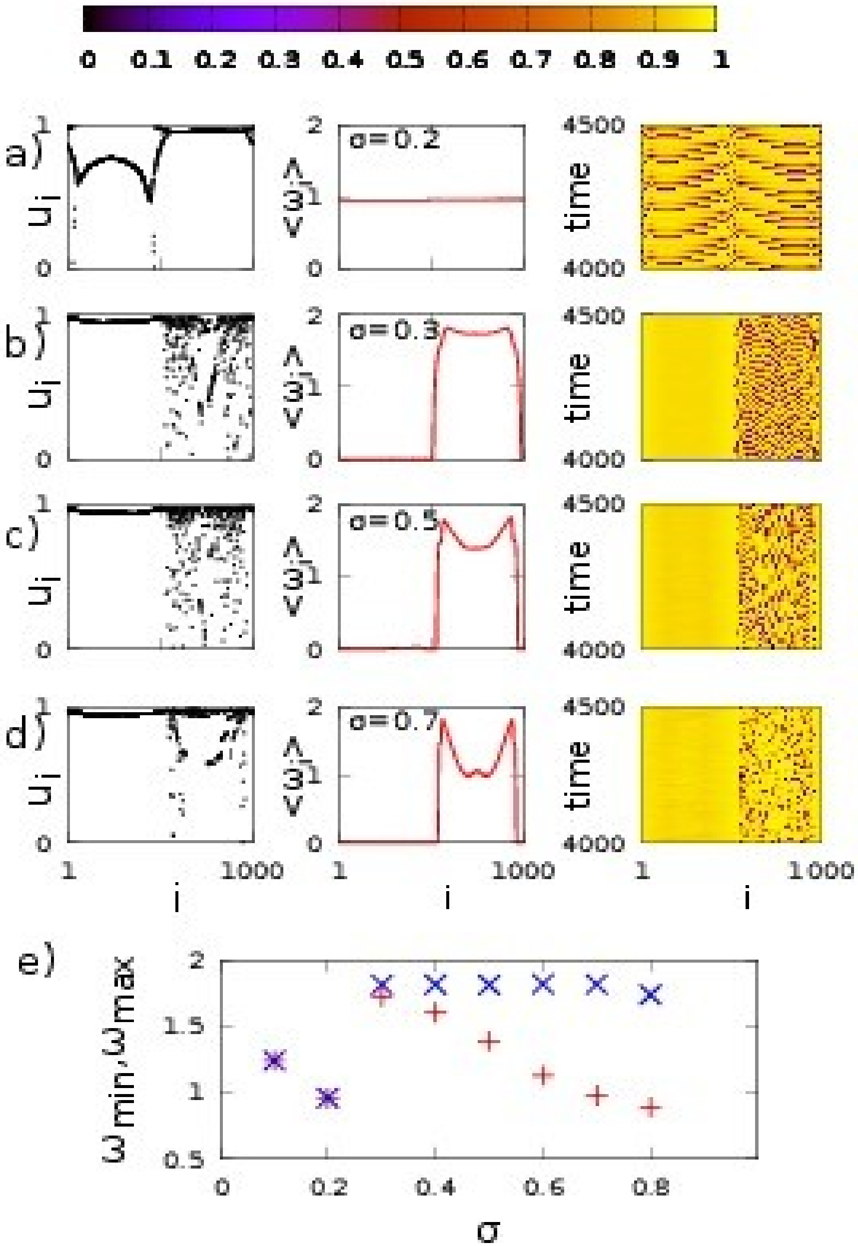




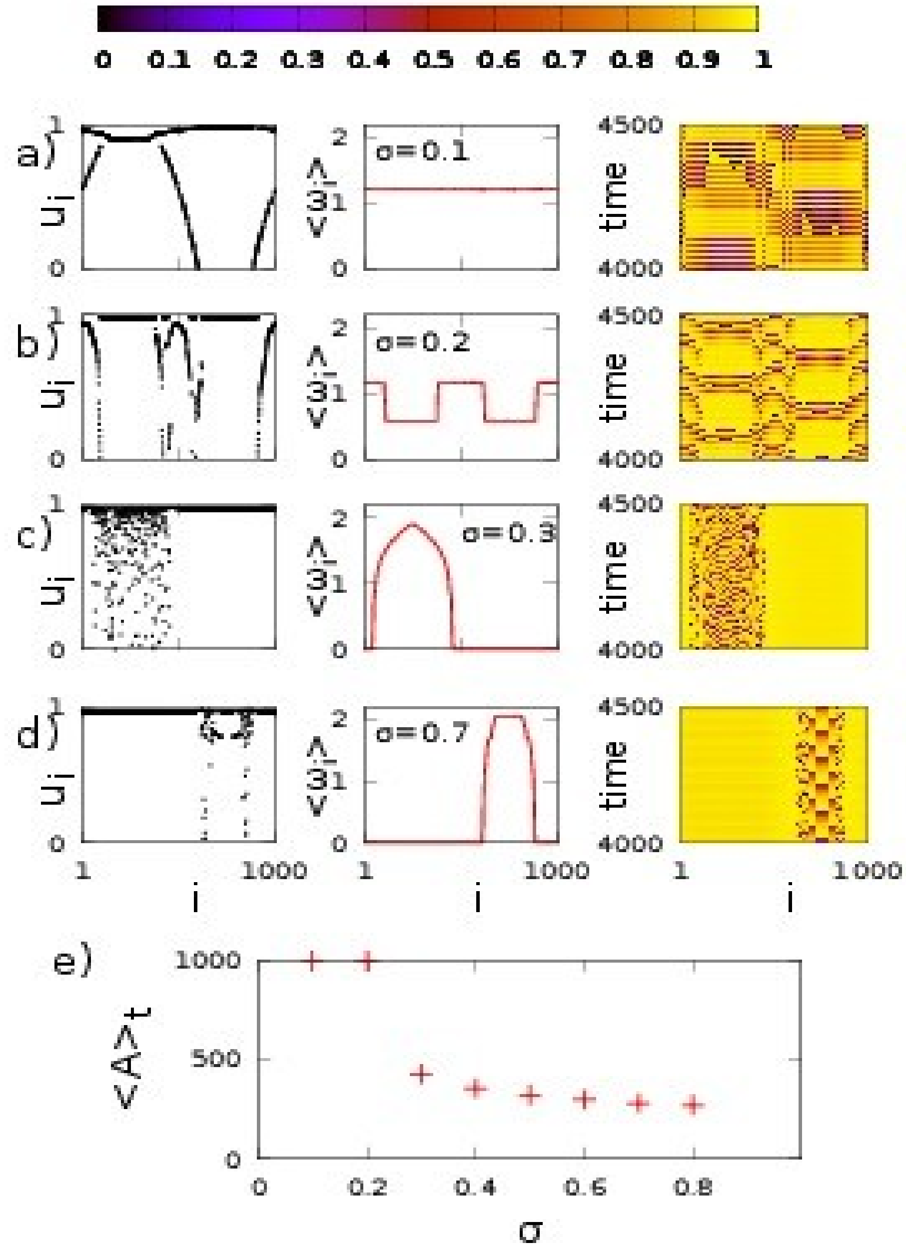
Confinement Phenomena: The activity gets confined in one semi-ring for small values of *R*. In the other semi-ring the elements stay near-threshold.

When $R \rightarrow N$ the activity extends to the entire system.

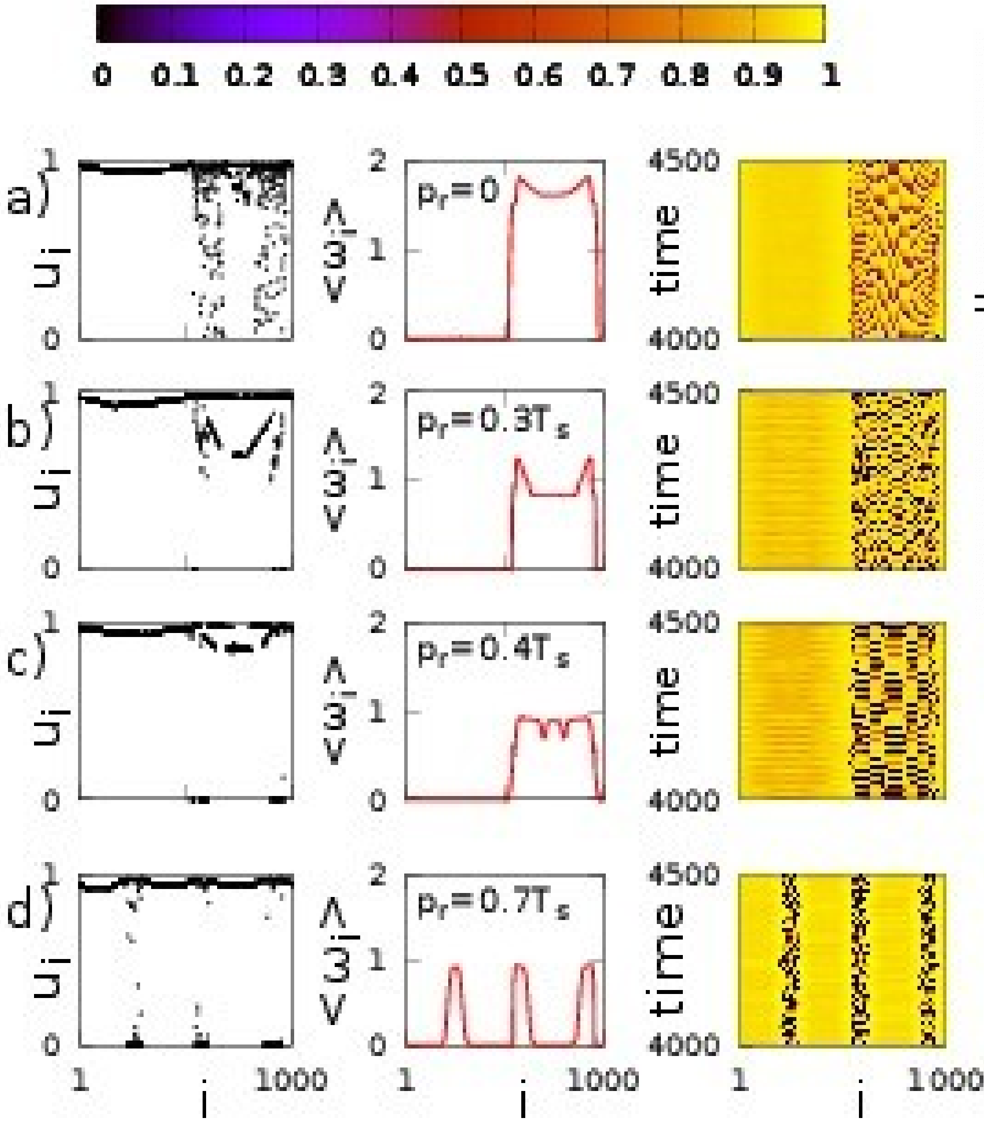
(σ=0.4, p_r=0, N=1000, μ=1.0)

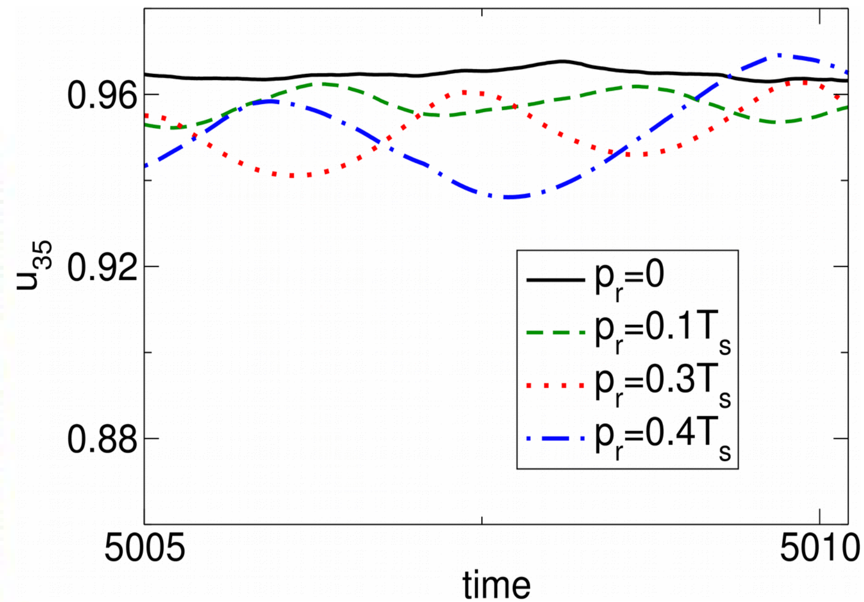

0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
a)
b)
c)
d)
e)
u_i
<ω_i>
time
σ=0.2
σ=0.3
σ=0.5
σ=0.7
4000
4500
1
1000
i
ω_min, ω_max
σ
0
0.2
0.4
0.6
0.8
0.5
1.5
2

(*R=100*)


0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
a)
b)
c)
d)
e)
u_i
<ω_i>
time
σ=0.1
σ=0.2
σ=0.3
σ=0.7
4000
4500
1
1000
i
<A>_t
σ
500
0
0.2
0.4
0.6
0.8

(R=300)

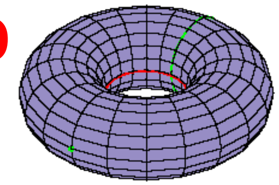


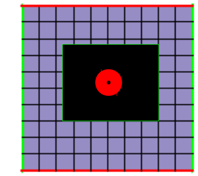


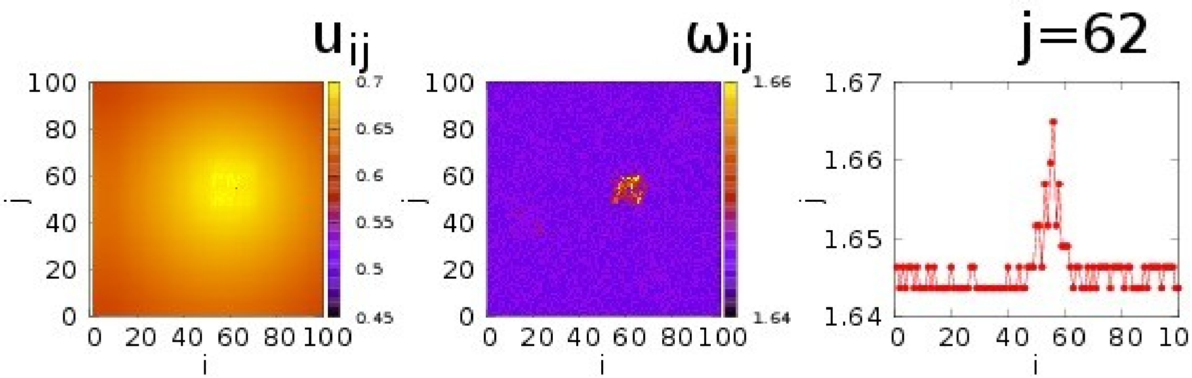
The near-threshold elements are not totally immobile, they perform short chaotic oscillations but stay near the threshold OR near their displaced fixed point!!!!.

$\sigma=\mathbf{0.4}$, $R=100$, $N=1000$, $\mu=1.0$ *and* $u_{th}=0.98$

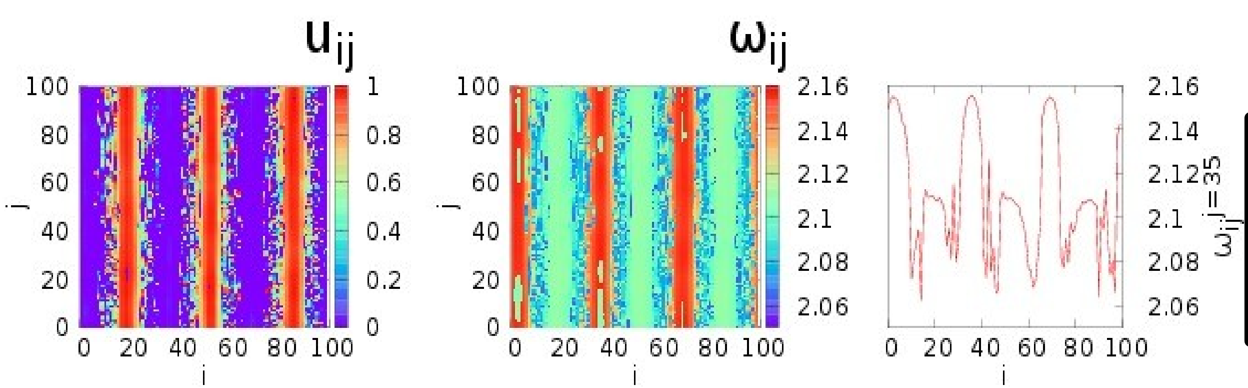
2.6. Nontrivial generalizations in 2D & 3D





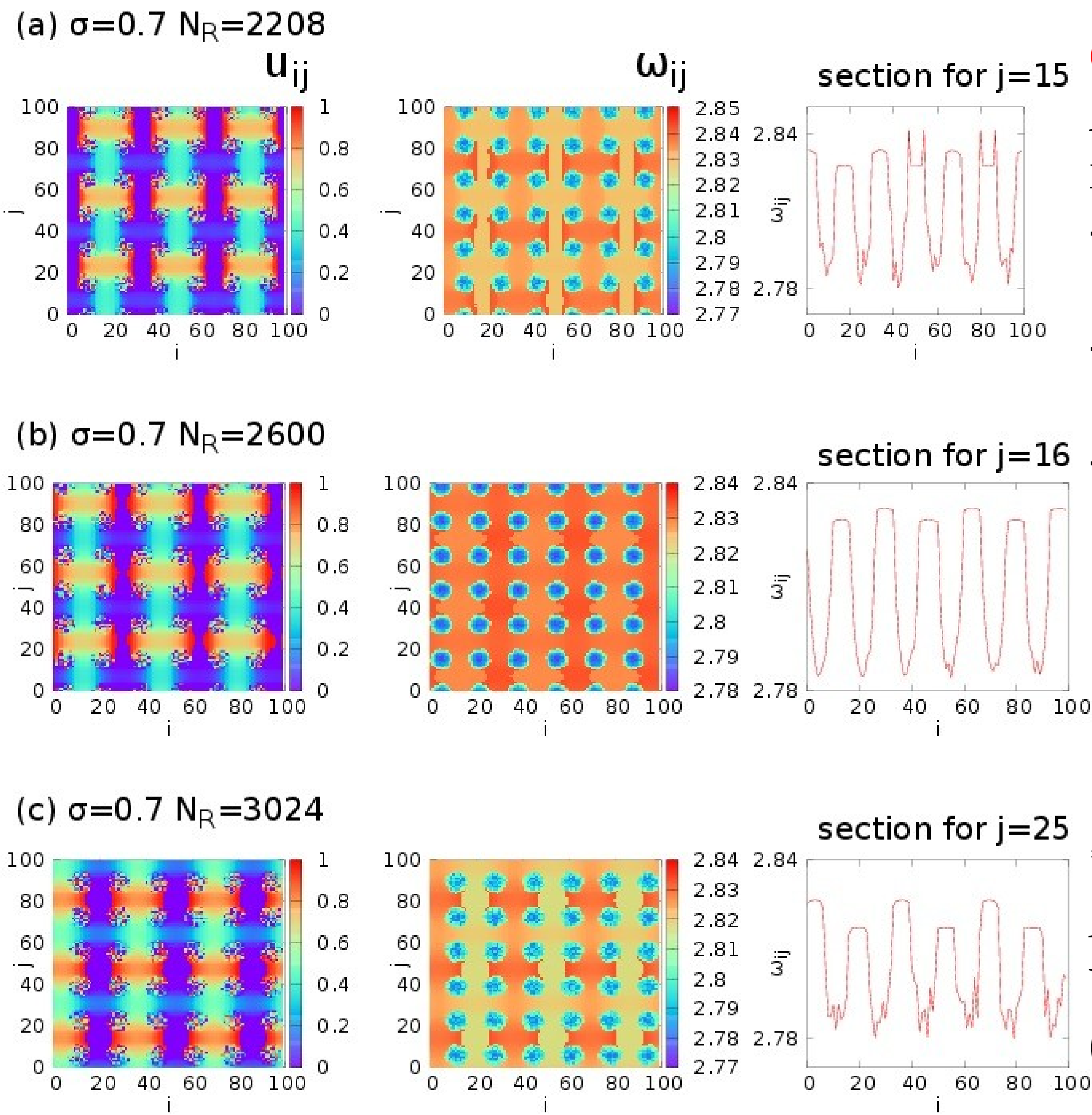


Direct generalization of 1D
(σ=0.1, R=10, p_r=0T_s)



Generalization of 1D but:
+ 2 coherent classes!
(σ=0.6, R=20, p_r=0.6T_s)

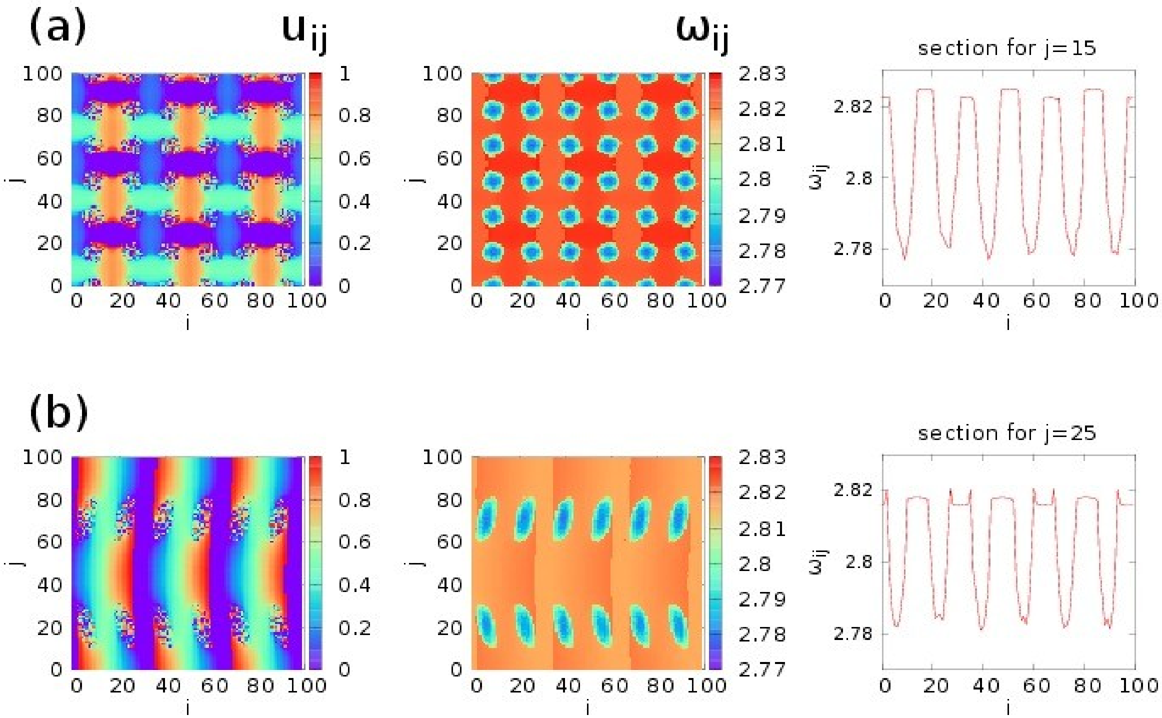
System size: N=100 X100, μ=1.0



Grid: cannot exist in 1D
As the region of interaction R increases:
- multiplicity does not change
- grouping of coherent regions takes place
- the groups become more distinct as R increases

$\sigma=0.7, \ p_r=0.22T_s$

*Only at $N_R \rightarrow$ system size We observe fewer (in)coherent regions (4 x4).



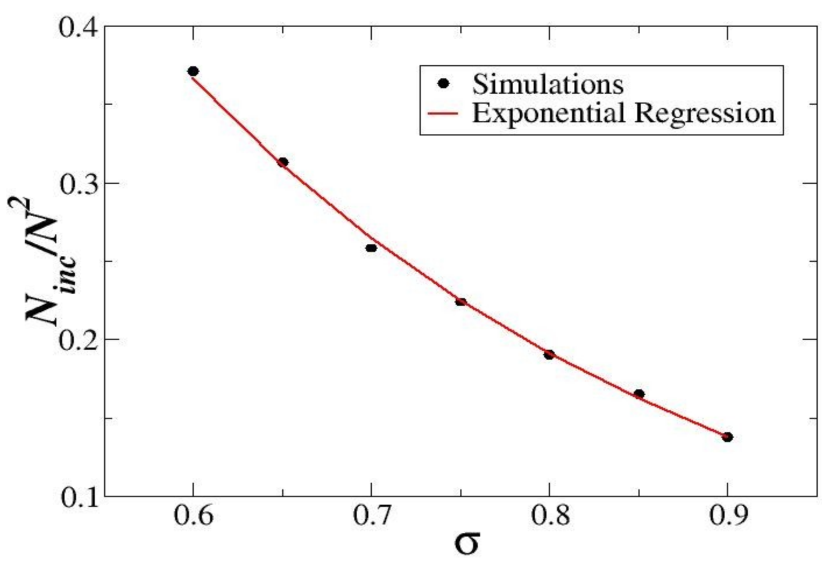
Bi-(multi)-stability!

$\sigma=0.7$, $p_r=0.22T_s$,

$R=22 \rightarrow N_R=2024$

Reversion of coherence

$\sigma \sim 0.5$

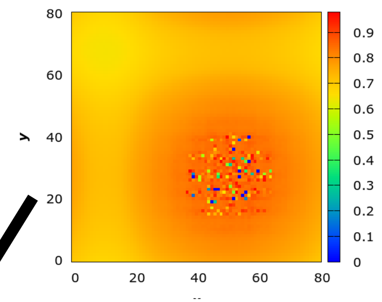


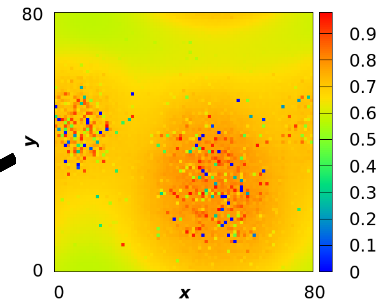
Coupling strength σ
Controls
- N_{inc}/N^2
and
- mean phase velocities

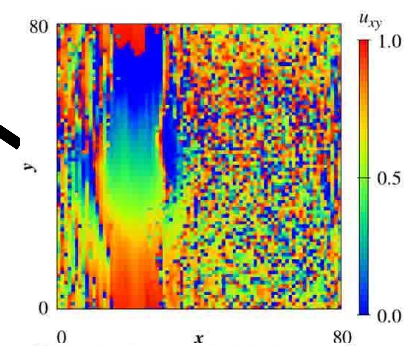
$p_r=0.22T_s$,
$R=25 \rightarrow N_R=2600$

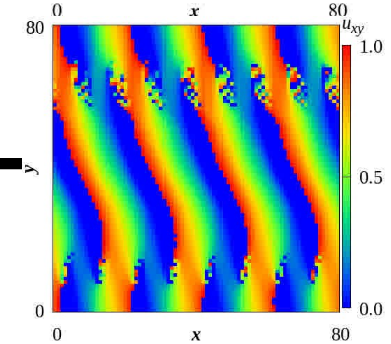
Map of 2D Chimera Patterns

sigma	Pattern
0.1	spot
0.2	spot
0.3	synchronized & double spot
0.4	synchronized & stripes
0.5	undefined/transition
0.6	undefined/transition
0.7	undefined/transition
0.8-1.4	grids & lines-of-spots

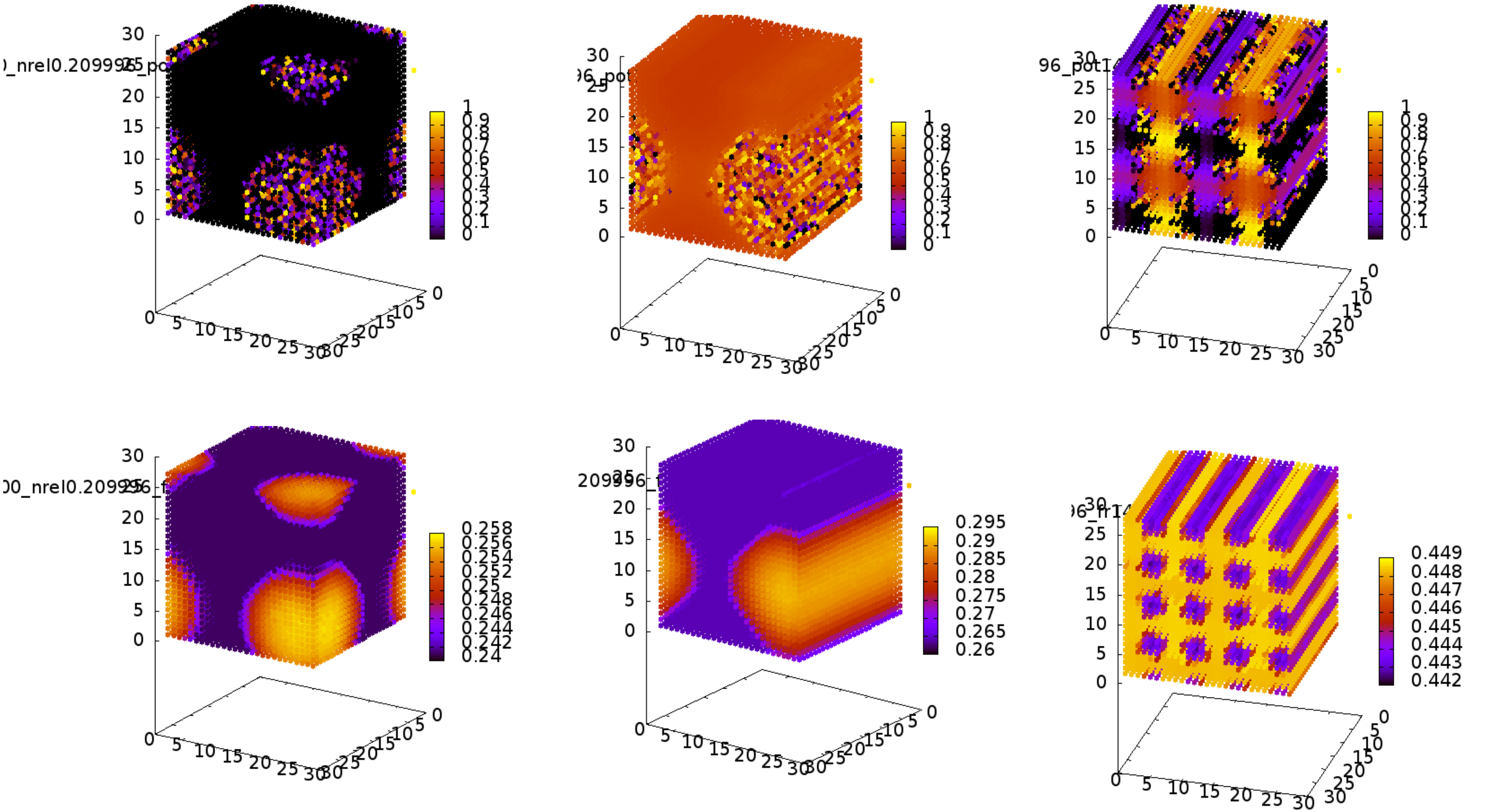








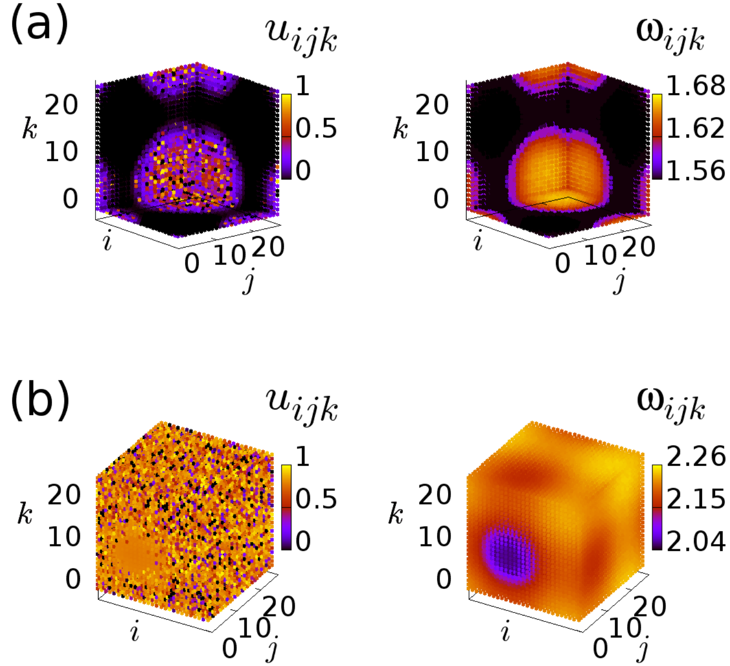
2.7 Results for chimeras in 3D connectivity

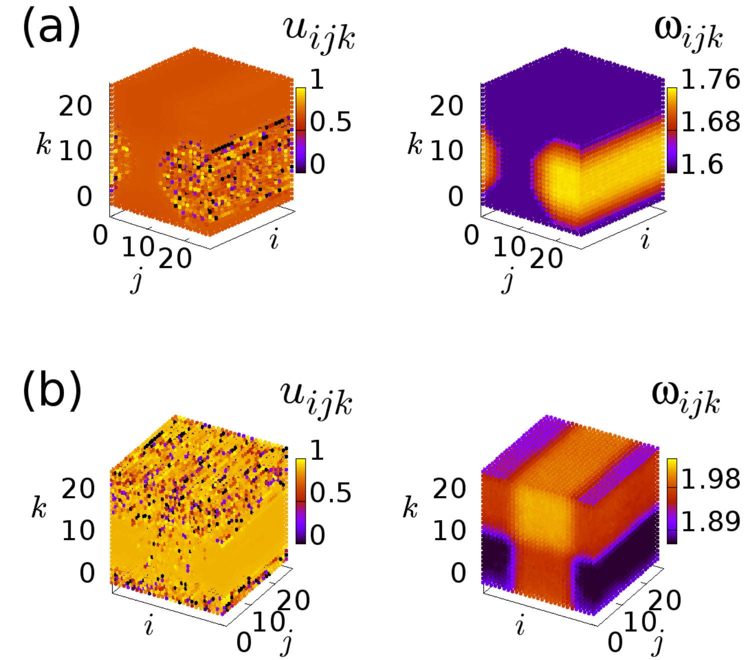


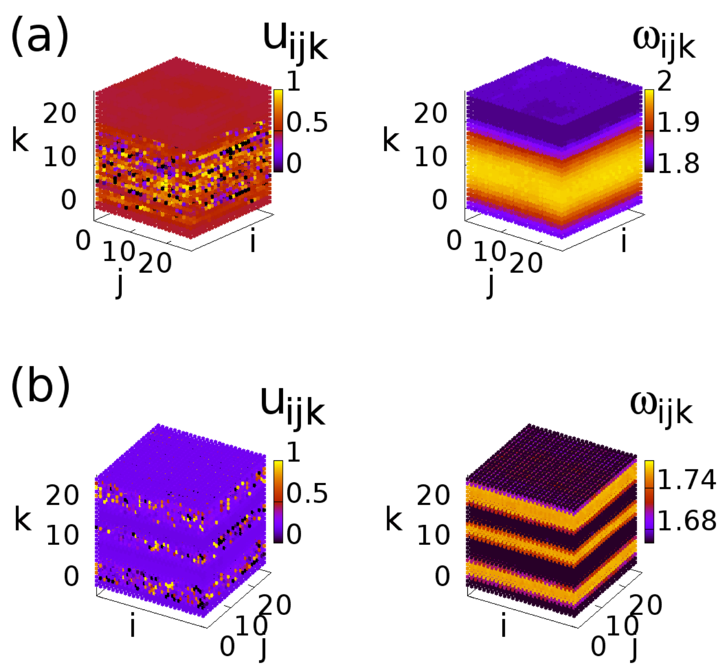
Incoherent spot	Incoherent cylinder	Grid
(p_r=0.47, σ=0.1)	(p_r=0.47, σ=0.2)	(p_r=0.61, σ=0.7)

Size: 27 X 27 X 27 = 20000; T=0.21Ts; top=potential, bottom=mean phase velocity

(*Y. Maistrenko et al., New Journal of Physics (2015): 3D-chimeras in Kuramoto model*)







Other 3D stable patterns

Reversion of coherence

σ~0.3

3.1 The FitzHugh Nagumo Model (1961):

[originates from the Hodgkin–Huxley model and models propagation of electrical signals in neurons]

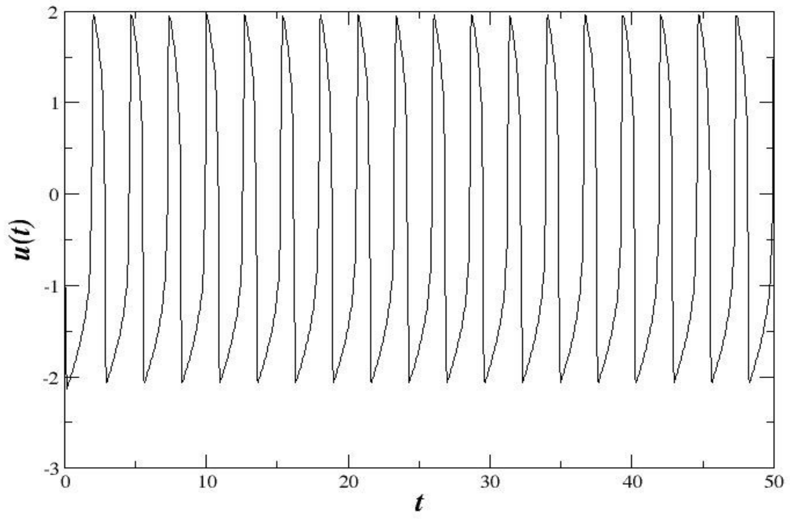
$$\epsilon \frac{du(t)}{dt} = u(t) - \frac{u^3(t)}{3} - v(t) + I(t)$$

$$\frac{dv(t)}{dt} = u(t) + \alpha$$

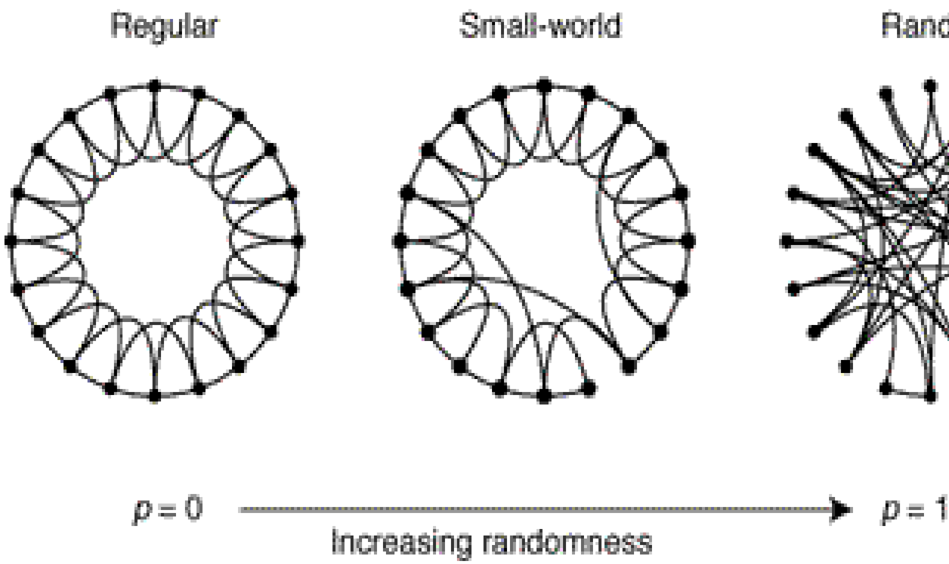
$\alpha = 0.5$
$\epsilon = 0.05$

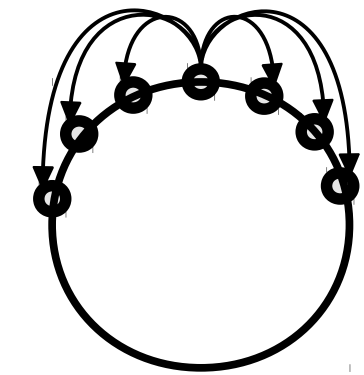
I(t)=const=0.5

u(t)=membrane potential (activator)
v(t)= recovery potential (inhibitor),
I(t)=external stimulus



3.2 Coupled FitzHugh Nagumo Oscillators (in a ring)



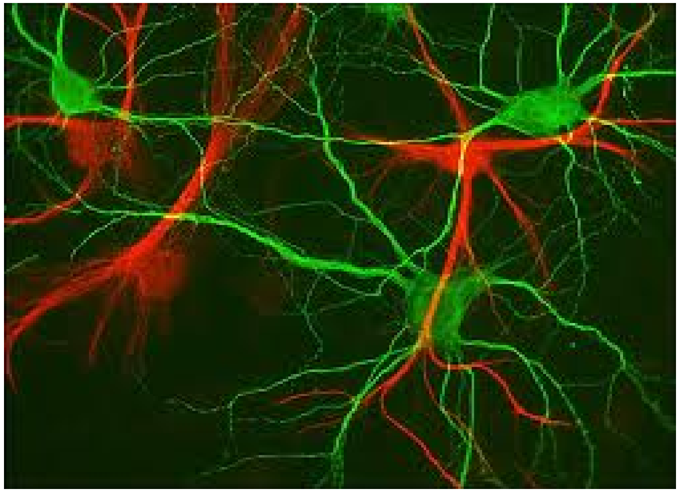


* With the current development on networks, a first approach is to put the oscillators in a ring

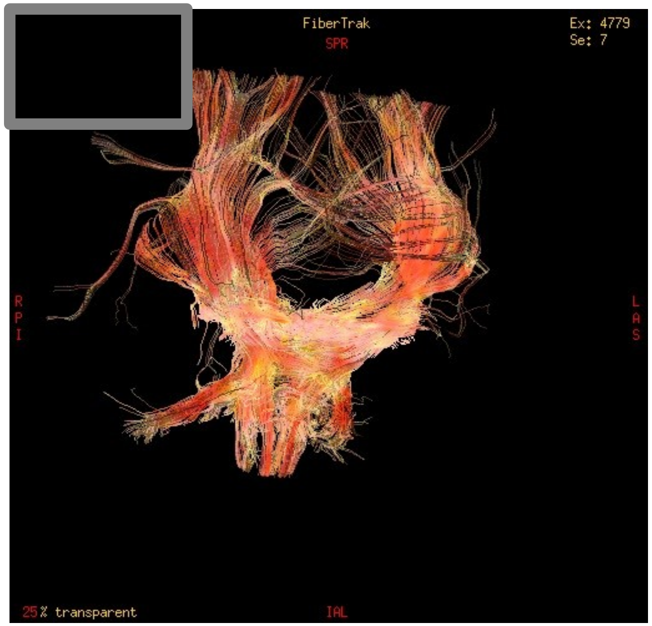
$$\epsilon \frac{du_i(t)}{dt} = u_i(t) - \frac{u_i^3(t)}{3} - v_i(t) + \frac{\sigma}{2R} \sum_{j=i-R}^{i+R} [u_j(t) - u_i(t)]$$

$$\frac{dv_i(t)}{dt} = u_i(t) + \alpha + \frac{\sigma}{2R} \sum_{j=i-R}^{i+R} [v_j(t) - v_i(t)]$$

[Parenthesis on Brain Connectivity:



Neurons: are electrically excitable cells which process and transmit information through electrical signals
- ***soma*** (contains the nucleus, typical 25μm)
- ***dendrites*** (receive signals)
- ***axons*** (connect neurons and transmit signals, size 1μm, max 1m!)
- ***axon terminals*** (contain synapses to communicate the signal)

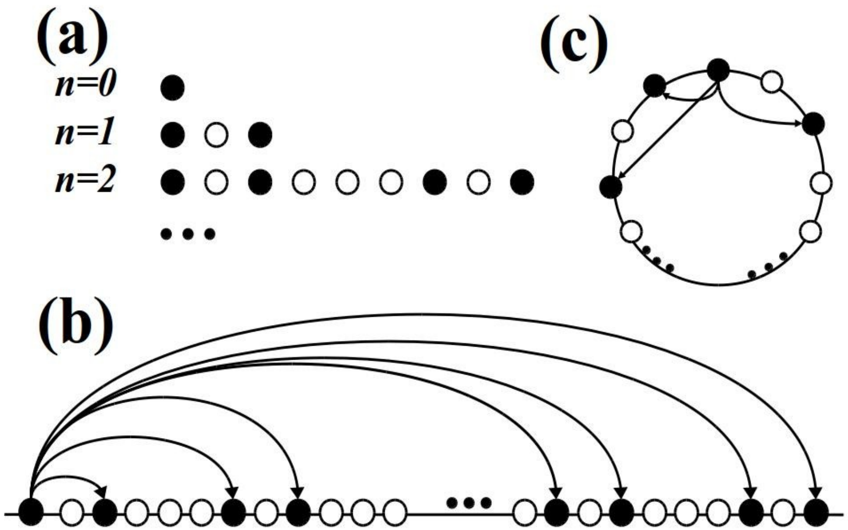


DTI – MRI: Neuron axons in 3D representation

- Resolution: 1-3mm
- Fractal dimensions of the neuron axons network: 2.5-2.6
- Different correlations and fractality for neurodegenerative disorders

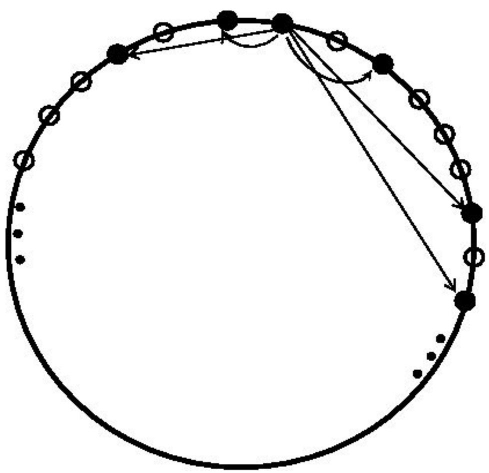
]

3.3 Coupling on Fractal Networks

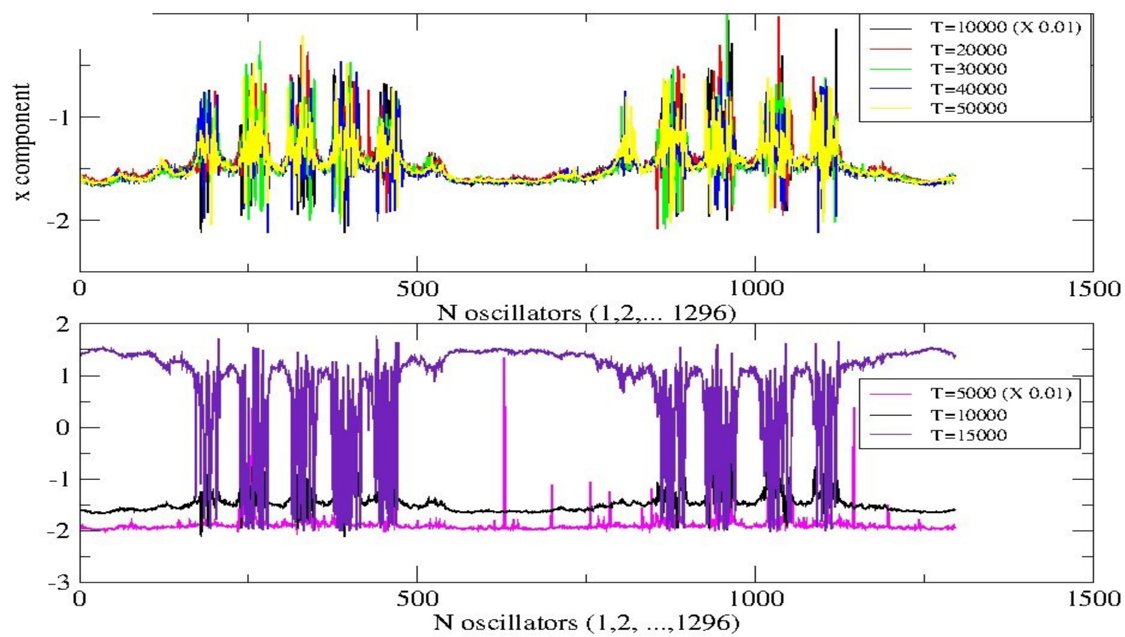


See: movie-fhn-fractal

Nested Chimera States



Random Fractal (2) connectivity ln4/ln6



Appearance and destruction of a nested/ramified/hierarchical chimera state

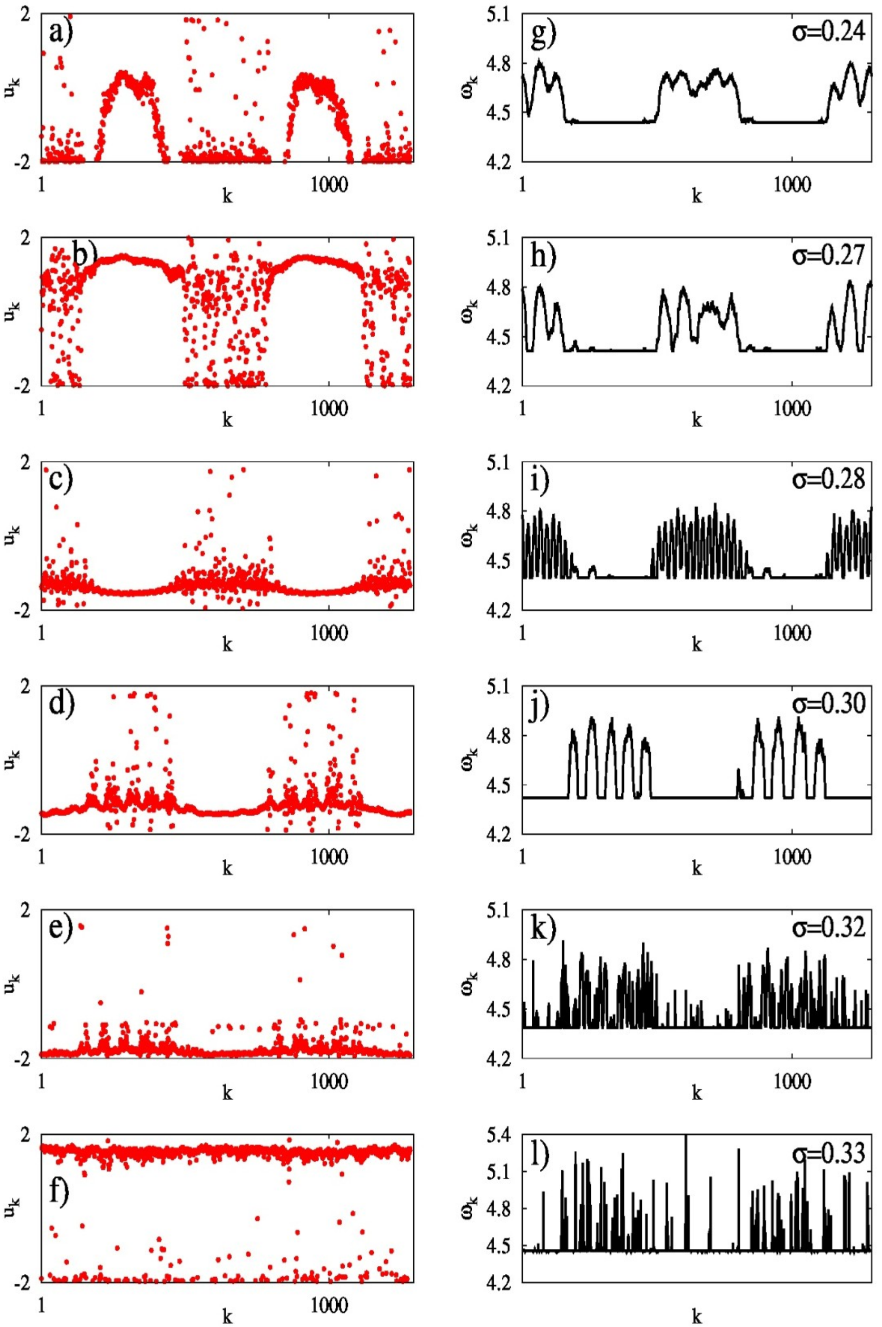
Ramifications are due to fractal connectivity

σ= coupling strength

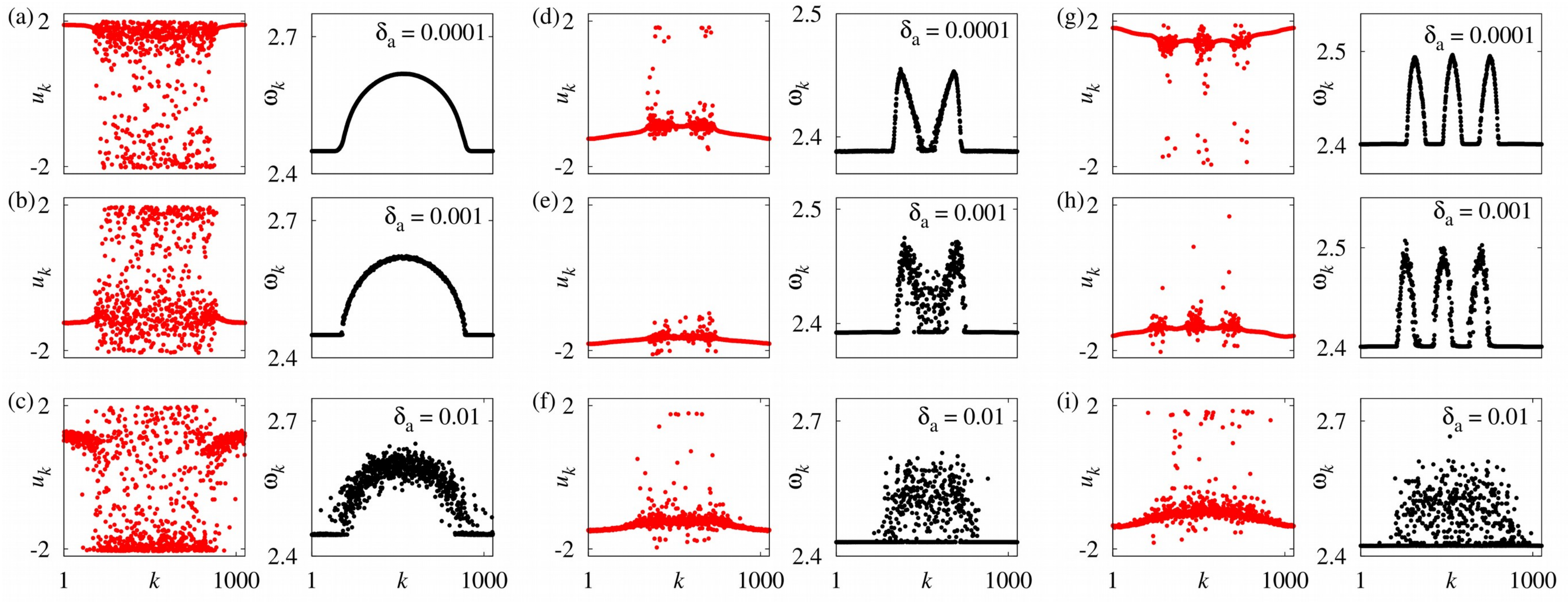
For σ>> we drive to synchronization

Omelchenko et al. PRE 2015

See movie: chimera-fractal



[Parenthesis: Effects of noise



Noise in the distribution of coupling strength σ

The role of spatial correlations in connectivity

I. Non-local connectivity
II. Asymmetric nonlocal
III. Fractal-hierarchical connectivity
IV. Reflecting connectivity
V. Diagonal connectivity
VI. Modular networks connectivity

→ Spatial correlations in connectivity

1. Random connectivity networks
2. Random values of the coupling strengths
3. Small world networks
…

4. Other realistic networks

→ If noise is added in the connectivity, chimera state starts disintegrating

4. Conclusions

- Chimera States in FHN and LIF neuron dynamics
- Spiking regime induces chimera states
- Nonlocal (spatially correlated) connectivity produces chimera states
- Hierarchical connectivity: traveling chimeras

Open Problems

- Connection of synchronization patterns with memory and cognition
- Interplay between topology and dynamics
- Spatial correlations in the connectivity => chimera states???

- Time dependent connectivity
- Apoptosis of neurons
- Influence of external forces on chimera states
- Influence of initial conditions...

Collaborations & Thanks

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* Prof. Eckehard Schoell

THANK YOU FOR YOUR ATTENTION !

Motivating Questions:

Theory:

- Why chimera numerical evidence is mostly linked with neuron-related models?

- Spiking dynamics essential in neuron models: Is it also essential for the production of chimera states?

- Role of connectivity and the formation of chimera states ?
  Are spatial correlations important for the formation of chimera states??

Applications:

- Are chimera states, as patterns formed under certain (external) conditions in co-operation with internal dynamics+connectivity, relevant in memory & cognition-related activities.
- Is the form of chimera patterns relevant in brain neurological/ neurodegenerative disorders?
- Can it be revealed in experiments of brain partial activity (such simple task Experiments: parroting, eye movement, finger tapping)?